POLITEKNIK MALAYSIA
SULTAN ABDUL HALIM MU'ADZAM SHAH

DISCRETE MATHEMATICS

Polytechnic edition

WORKBOOK

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MATHEMATICS, SCIENCE, AND COMPUTER DEPARTMENT

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Polytechnic edition

WORKBOOK

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PREFACE

Alhamdulillah praise to be Allah S.W.T, with His grace and mercy; peace and blessing upon Prophet Muhammad P.B.U.H. his family and companions; this workbook has been completed.

The content of this workbook has been designs to meet the syllabus requirements of polytechnics so that students can make use out of it. The process of completing the workbook smooths along the way since we were experienced in teaching and learning of this course for several years

This workbook caries synopsis notes and exercises from 5 chapter. Every chapters introduces a synopsis of notes and is augmented by details worked examples that lead on to the questions for the exercises attached in the end of the section.

We hope to see this workbook as a catalyst to attracted interest students to go on learning more related advanced material such as Discrete Mathematics and Its Applications course.

CONTENTS

CHAPTER 1

Sets, Relations & Function

EQUAL SETS

The set are equal if and only if their number of elements and the member of elements are exactly same.

Example of equal sets: **A = {5,6,7}, B = {7,5,6}, C = {5,5,6,6,7,7}**

SPECIAL SYMBOLS FOR SETS

N = the set of **positive integers** : 1, 2, 3, ….

Z = the set of **integers** : ….., -2, -1, 0, 1, 2, …..

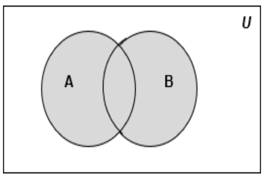
Q = the set of **rational numbers**

R = the set of **real numbers**

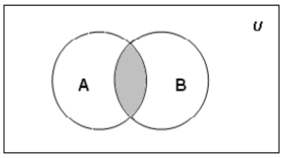
C = the set of **complex numbers**

OPERATION ON SETS

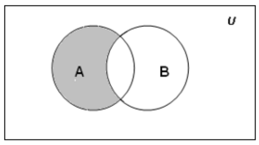
$A \cup B$



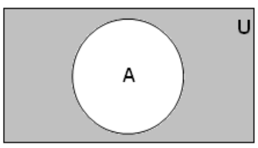
$A \cap B$



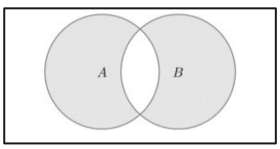
$A - B$



A'



$A \oplus B$

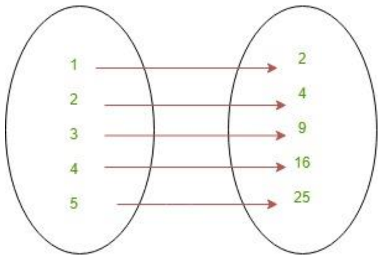


RELATION REPRESENTATION

GRAPHICALLY/ ARROW DIAGRAM

Example:

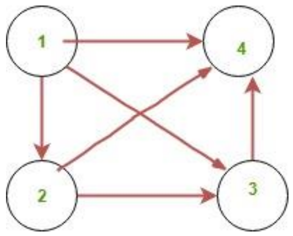
R= {(1, 1), (2, 4), (3, 9), (4, 16), (5, 25)}



DIGRAPH (DIRECTED GRAPH)

Example:

R= {(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}



MATRICES

Example:

Suppose that A = {1,2,3} and B = {1,2}. R= {(2, 1), (3,1), (3,2)}

In matrix form;

$$\boldsymbol{M_R} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} \end{bmatrix}$$

PROPERTIES OF RELATIONS

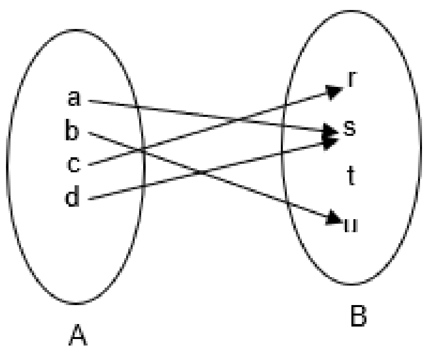
- **REFLEXIVE**
- **SYMMETRIC**
- **TRANSITIVE**

*A relation on a set A is called an **equivalence relation** if it is *reflexive*, *symmetric* and *transitive*.

IMPORTANT TERMS USED IN FUNCTIONS

- The element in set A is called the ***domain***
- The element in set B is called ***codomain***
- Unique element of B which is assign to A is called ***image / range.***

Example 1:



Domain = {a, b, c, d}

Codomain = {r, s, t, u}

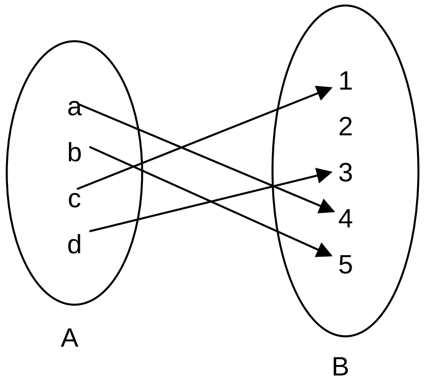
Range/Image = {r, s, u}

ONE-TO-ONE FUNCTIONS

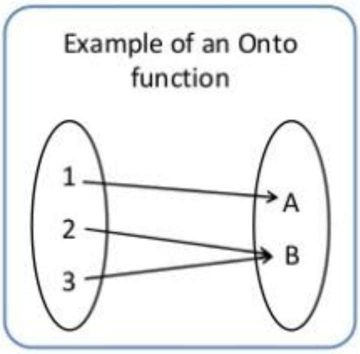
Example 2:

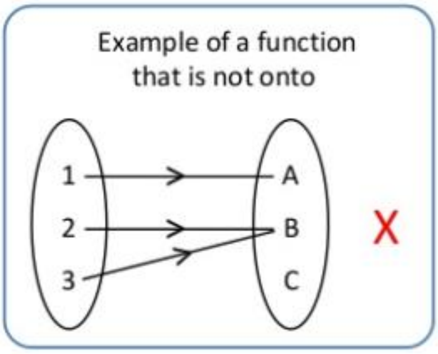
Determine whether the function f from {a, b, c, d} to {1, 2, 3, 4, 5} with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$ and $f(d) = 3$ is one-to-one.

Solution: The function f is one-to-one.



ONTO FUNCTION





EXERCISE 1A

1. List the elements of the following sets; here $N = \{1,\ 2,\ 3 \ldots\}$.

a) $A = \{x : x \in N,\ 3 < x < 10\}$

b) $B = \{x : x \in N,\ x\ is\ even,\ x < 15\}$

c) $C = \{x : x \in N,\ 4 + x = 7\}$

d) $D = \{x : x \in N, x\ is\ odd\ number, x < 20\}$

e) $\mathrm{E} = \{x : x \in N, x$ is a factor of $20, x \leq 20\}$

2. Given the universal set) $U = \{x : 15 \leq x \leq 26,\ x\ is\ an\ integer\}$,

$$M = \{x : x\ is\ a\ prime\ number\};$$

$$N = \{x : x\ is\ a\ multiple\ of\ 3\};$$

$$O = \{x : x\ is\ an\ even\ number\}$$

a) List the elements of set M, N and O.

b) List the elements of set N' and O' .

c) State the value of *n (N')* and *n (O')*.

3. List the element of the following sets:
   a) $\{x: x \in N, 5 < x < 12\}$, where $N = \{1, 2, 3\ldots\}$

   b) $\{x: x \in N$, x is even, $x < 15\}$, where $N = \{1, 2, 3\ldots\}$

   c) $\{x: x \in N, 10 < x < 35$, x with sum of digits less than 6$\}$ where $N = \{1, 2, 3\ldots\}$

4. Given that the universal set .

$$\xi = \{x : x \text{ is an integer}, 1 \le x \le 10\}$$
$$P = \{x : x \text{ is an odd number}\}$$
$$Q = \{x : x \text{ is a factor of } 12\}$$

   a) List the element of set P and set Q

   b) State the value of $n(P)$ and $n(Q)$

5. List all the elements of the following sets:
   a) A= {x: x is a letter in the word MATHEMATICS}

   b) B= {x: x is a month of a year not having 31 days}

   c) C= {x: x is a letter before e in the English alphabet}

   d) D= {x: x is a vowel in the word “EQUATION”}

6. Determine whether each pair of sets is equal
   a) {1, 2, 2, 4}, {1, 2, 4}

   b) {1, 1, 3}, {3, 3, 1}

   c) $\{x \mid x^2 + x = 2\}$, {1, -1}

   d) $\{x \mid x \in \mathrm{R} \text{ and } 0 < x \leq 2\}$, {1, 2}

EXERCISE B

1. Consider the following sets:

   $\emptyset$, $A = \{1\}$, $B = \{1, 3\}$, $C = \{1, 5, 9\}$, $D = \{1, 2, 3, 4, 5\}$,

   $E = \{1, 3, 5, 7, 9\}$, $U = \{1, 2, \ldots\ldots, 8, 9\}$

   Insert the correct symbol ⊆ or ⊈ between each pair of set.

   a) $\emptyset$, A b) A , B c) B , C d) B , E

   e) C , D f) C , E g) D , E h) D , U

2. Use a Venn Diagram to illustrate the relationship
   a) $A \subseteq B$ and $B \subseteq C$.

b) $A \subseteq D$ and D$\subseteq U$

3. The universal set,

$$U = \{x: 10 < x < 35, x \text{ is an integer}\},$$

$$F = \{x: x \text{ is a prime number}\},$$

$$G = \{x: x \text{ is a multiple of } 3\}$$

$$H = \{x: x < 20\}.$$

a) List all the elements of set F, G and H.

b) Draw the Venn diagram to represent all elements of **F ∪ G ∪ H** in the universal set.

c) Find **n(F ∩ H)**

EXERCISE C

1. Given $A = \{1, 2, 3, 4, 5\}$, $B = \{4, 5, 6, 7\}$, $C = \{5, 6, 7, 8, 9\}$,

   $D = \{1, 3, 5, 7, 9\}$, $E = \{2, 4, 6, 8\}$, $F = \{1, 5, 9\}$.

   Find:

   a) $A \cup B$

   b) $B \cap D$

   c) $D \cap E$

   d) $F \backslash A$

   e) $C \cup F$

   f) $E - B$

   g) $D \backslash A$

   h) $B \oplus C$

2. Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find:

   a) $A \cup B$

   b) $A \cap B$

   c) $A - B$

   d) $B \oplus A$

3. Given $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{2,4,6,8\}, B = \{1,3,4,5,7\}$, $C = \{7,8\}$

   a) $A \cap B$

   b) B'

   c) $A' \cup B'$

   d) $n(A - B)$

   e) $(A - C)'$

   f) $(A \cap C) \cap (A \cup B)$

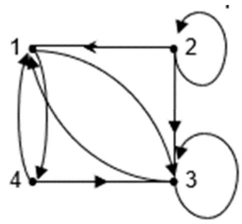
4. Given A={x:x is an integer and-3≤x<7) and B={x:x is a natural number less than 6)

   a) List all the elements of the set A and B

   b) Find A⊕B

EXERCISE D

1. What are the ordered pairs in the relation R represented by the directed graph shown below?



2. Draw a directed graph (Digraph) for the relation

   a) R = { (1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)} on the set {1, 2, 3, 4}

   b) R = {(a,b), (a,a), (b,b), (b,c), (c,c), (c,b), (c,a)} on the set {a, b, c}

3. **List** and display all the relation **graphically** the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3} where (a, b) $\in$ R if and only if

   a) a = b

b) $a + b = 4$

c) $a > b$

d) a divides b $(a \mid b)$ *(*it means b / a)*

State the **domain and range** for all the questions above.

EXERCISE E

1. Consider the relation on the set { 1, 2, 3, 4, 5}:R = { (1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5) }, determine whether the relation R is reflexive, symmetric or transitive. Explain your answer.

2. Let M = {0, 1, 2,3} and defined relation R = {(0,1), (0,3), (1,0), (1,1), (2,3), (3,0), (3,2), (3,3)}

    a) Represent the relation R using directed graph

    b) Determine whether the relation R is reflexive, symmetric or transitive. Explain your answer.

3. Given those three relations on set A = {1,2,3,4}:

    R = {(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)}

    S = {(1,1),(1,3),(1,4),(3,4)}

    T = {(1,2),(2,2),(2,3)}

    a) Determine which relations are reflexive. Give your reason.

    b) Determine which relations are not symmetrical. Explain your answer.

    c) Give a reason why S is transitive.

4. Given A = {1,2,34}, B = {1,4,6,8,9} where element a is in A is related to element b in B, if and only if b = a^2

   a) List the element of the relation

   b) Draw the directed graph for the relation

   c) Determine whether the relation is reflexive or not.

   d) Is the relation symmetric? Explain your answer.

EXERCISE F

1. Given that $f(x) = 3x + x^2$, $g(x) = \frac{x}{2} - 4$ and $h(x) = x - 5$. Determine,

   a) $\mathrm{fh}(\mathrm{x})$

   b) gh(2)

   c) $\mathrm{h}^2(6)$

2. Given $f(x) = 4x - 1$ and $g(x) = x^2 + 3$, find
   a) $\mathrm{fg}(-3)$

   b) The value of x when $\mathrm{f}^2(\mathrm{x}) = 7$

3. Given that f(x)= $2x+x^2$ and g(x)= $1 - \frac{x}{4}$
   a) f(3)

   b) fg(– 4)

4. Let f and g be functions from the positive integers to the positive integers defined by equations f(n) = 2n + 1, g(n) = 3n – 1 . Find the compositions f ∘ f , g ∘ g , f ∘ g , and g ∘ f.

5. The function *f* and the composite function *gf* are defined as follows. Find the function *g*.

   a) $\mathrm{f(x) = x + 1,\ gf(x) = \frac{3}{x-2}, x \neq 2}$

   b) $\mathrm{f(x) = 3x + 2,\ gf(x) = 9x^2 + 9x + 2}$

6. Given the functions $f(x) = 3x + 7$ and $fg(x) = 22 - 3x$, find $gf(x)$.

EXERCISE G

1. Given g = {(1,b), (2,c), (3,a)}, a function from X = {1, 2, 3} to Y = {a, b, c, d} and

   f = {(a,x), (b,x), (c,z), (d,w)}, a function from Y to Z = {w, x, y, z}, write $f \circ g$ as a set of ordered pairs and draw the arrow diagram of $f \circ g$.

2. Given that the function f(x) = 4x + 1, find a formula for $f^{-1}(x)$.

3. Given that the functions g(x) = x – 2 and f (g(x)) = $x^2 - 4x + 8$. Find:
   a) g (8)

   b) The values of x if f (g(x)) = 13

   c) The function f

   d) $g^{-1}(-1)$

4. Let $f(x)= 2x+1$ and $g(x) = 3x^2 - 4$. Find:
   a) $(f \circ g)(x)$

   b) $(g \circ f)(-2)$

   c) $f^{-1}(x)$

5. The function f and g are defined by f:⟼$3x$, $g(x)$↦x+2
   a) Find an expression for $(g \circ f)(x)$

   b) Show that $f^{-1}(18)$+ $g^{-1}(18)$=22

6. The information below defines the functions h and g

$h: x \to 2x - 3$
$g: x \to 4x - 1$

Find $gh^{-1}(x)$

ADDITIONAL EXERCISE

1. Given $\xi = P \cup Q \cup R$ where

$$\xi = \{x : 3 \leq x \leq 10\}$$
$$P = \{x : x \text{ is a prime number}\}$$
$$P \cup Q = \{x : x \text{ is an odd number}\} \text{ and } P \subset Q$$

   a) list the elements of:

      i) set Q

      ii) set $Q' \cup P$

   b) find $n(P' \cap Q)$

2. Given the universal set U = {a, b, c, d, e, f, g, h}, set R = {a, b, c, d, e, f}, set S = {c, d, e} and set T = {f, g, h}.

   a) Construct a Venn diagram illustrating the sets.

   b) What is the relation between set R and set S?

   c) Give the element for S’ U T.

   d) Find n(R∩T’).

3. Given that the functions $g(x) = x - 2$ and $fg(x) = x^2 - 4x + 32$. Find

   a) The values of x if $g(x) = 20$

   b) The function f

   c) $g^{-1}(6)$

4. Given the function $f(x) = 6x + 4$ and $g(x) = 2x^2 + 3x$.
   a) Calculate $f(4)$.

   b) Determine $fg(x)$ and $gf(x)$.

   c) Determine $f^{-1}(5)$.

5. Let $A = \{a, b, c, d\}$ and $B = \{1,3,5\}$.Given that the function $f = \{(a, 1), (b, 3), (c, 5), (d, 5)\}$.
   a) Sketch the arrow diagram of the function *f.*

   b) State whether the function *f* is one to one or onto function.

6. Given the function $f(x) = x - 5$ and $g(x) = 3x^2 - 2$. Determine:

    a) The value of $g(2)$

    b) The value of $fg(x) = 5$

7. Given the function $g(x) = 5x + 7$ and $gf(x) = 7x - 9$. Determine:

a) The value of x if $g(x) = -8$

b) The function of $f(x)$

c) $g^{-1}(3)$

8. Given that the universal set $\xi = \{x: 12 \leq x \leq 25, x \text{ is an integer}\}$,

Set $P = \{13,15,16,18\}$,

Set $Q = \{x: x \text{ is a prime number}\}$ and

Set $R = \{x: x \text{ is and odd number}\}$.

a) Find the element for the set $(\mathrm{P} \cup \mathrm{Q})' \cap \mathrm{R}$.

b) Sketch the Venn diagram for the set ξ, P, Q and R.

9. Given the function $g(x) = \sqrt{(x-1)}$ and $f(x) = x^2 + 1$.

   a) Calculate $f(2) - g(17)$.

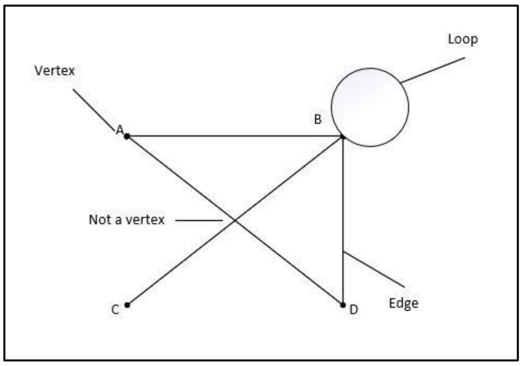
   b) Determine $gf(x)$.

   c) Determine $f^{-1}(5)$.

CHAPTER 2

Directed Graph

GRAPH TERMINOLOGY



SUMMARY OF GRAPH TERMINOLOGY IN GRAPH THEORY

The **size** of a graph is the number of its edges.

The **degree of a vertex** written $deg(v)$ is equal to the ***number of edges*** which are incident on v

The **sum of the degrees** of the vertices of a graph is equal to ***twice the number of edges***

The vertex v is said to be **even or odd (parity)** according as $deg(v)$ is even or odd

A vertex v is **isolated** if it is does not belong to any edge

A vertex with degree 1 is called a **leaf vertex**

The incident edge of vertex with degree 1 is referred as a **pendant edge**

A **path** is the sequence of connected vertices.

A **simple path** is a path where the vertices are only passed through once

A **trail** is a path where each edge is traveled once, meaning that there are no repeated edges (all edges are distinct)

The **length** of the path is the number n of edges that it contains

The **distance** between two vertices is described by the length of the shortest path that joins them

A **cycle / simple cycle** is a closed path with at least 3 edges, and no repeated vertices and

An **acyclic** is a graph that has no cycles in it

A **closed path or circuit** is a path that starts and ends at the same vertex

A graph is called **planar** if it can be drawn in the plane without any edges crossing each other

EULER PATH, EULER CIRCUIT, HAMILTON PATH, HAMILTON CIRCUIT

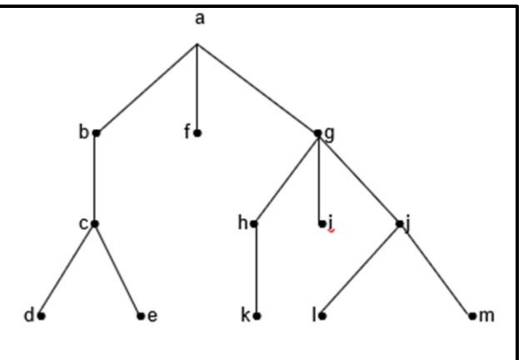
Euler path -A connected multigraph has an **Euler path** if and only if it has **exactly two vertices of odd degree**.

Euler Circuit- A connected multigraph has an **Euler circuit** if and only if **every vertex have even degree.**

Hamilton path-A **Hamilton path** is a simple path in a graph G that **passes through every vertex exactly once.**

Hamilton Circuit-A **Hamilton circuit** is a simple circuit in a graph G that **passes through every vertex exactly once.**

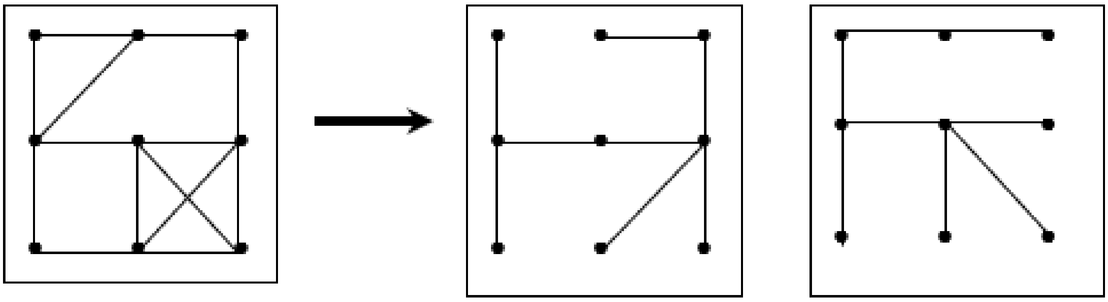
EXAMPLE OF TREES



A Rooted Tree

- The **root** is **a**.
- The **parent of h,i** and **j** is **g**.
- The **children of b** is **c.** The **children of j** are **l** and **m.**
- **h, i** and **j** are a **sibling**.
- The **ancestor of e** are **c, b** and **a**.
- The **descendants** of **b** are **c, d** and **e.**
- The **internal vertices** are **a, b, c, g, h** and **j.** (vertices that have children)
- The **leaves** are **d, e, f, i, k, l** and **m.** (vertices that have no children)

SPANNING TREES



MINIMAL SPANNING TREES

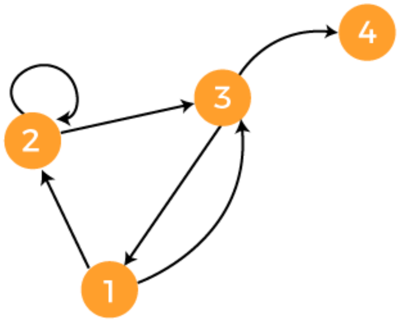
PRIM'S ALGORITHM

KRUSKAL ALGORITHM

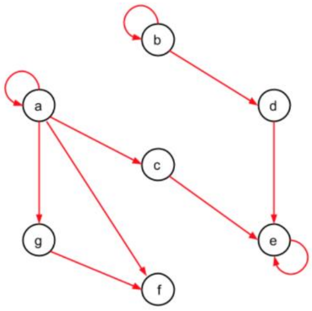
EXERCISE A

1. Find the **number of vertices**, the **number of edges**; identify all **parallel edges**, **loops** and **isolated vertices** for the following graph.

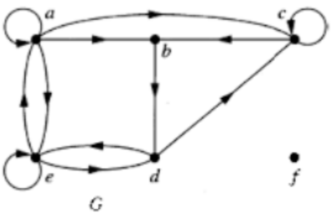
   a)



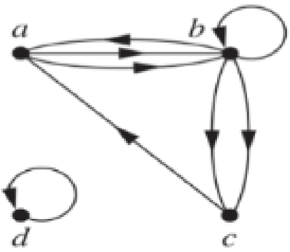
   b)



   c)

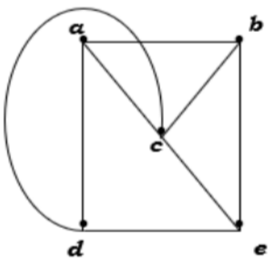


   d)



EXERCISE 2B

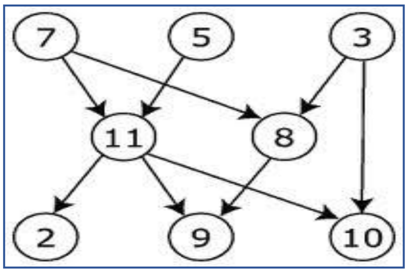
1. Find the degree of each vertex for the following graph.



2. Draw a graph having the given properties or explain why no such graph exists.
   a) Six vertices each of degree 3

   b) Five vertices each of degree 2

3. From the graph shown below,



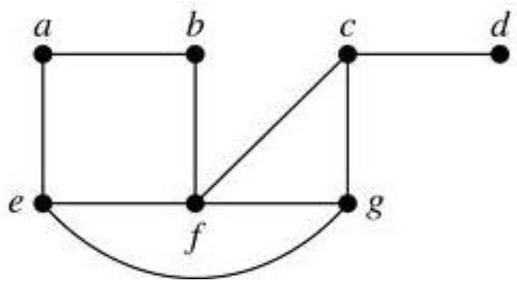
   a) List down in a table form the in degree and the out degree of each vertices

b) Determine the parity (even or odd) for each vertex and

c) Determine the sum of degrees

EXERCISE C

From the graph below,



a) Determine the **parity** (even or odd) **for each vertex**

b) Identify the **leaf vertex**

c) Identify the **pendant edge**

d) Identify the **distance**, D from *a* to *g*

e) Identify the **size** of the graph

f) Find the 2-simple **path**, P_1, P_2 from *a* to *c*

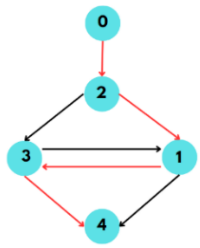
g) Find a **trail**, T from *d* to *e*

h) Find 3 **simple cycle**, labeled as C_1, C_2 and C_3

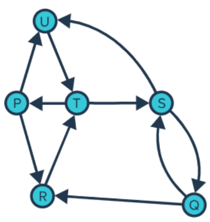
EXERCISE D

1. Base on diagram below, does **Hamilton path** or **Hamilton circuit** exist? If any, list down the Hamilton path or Hamilton circuit.

   a)



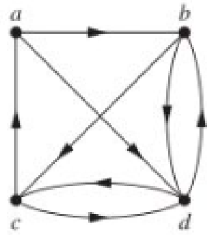
   b)

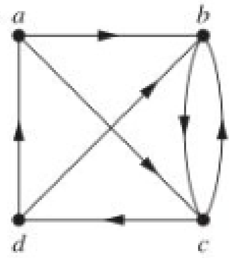


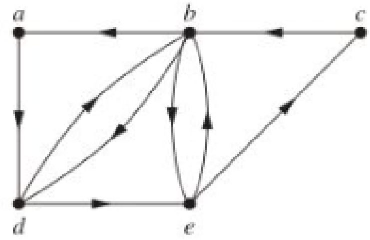
EXERCISE E

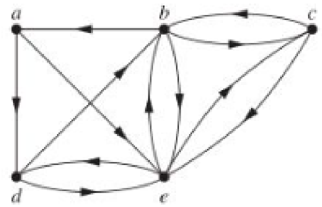
1. Give the characteristic of Euler path and Euler circuit.

2. Which of the following graphs has Euler path or Euler circuit? If any, list down the Euler path or Euler circuit.

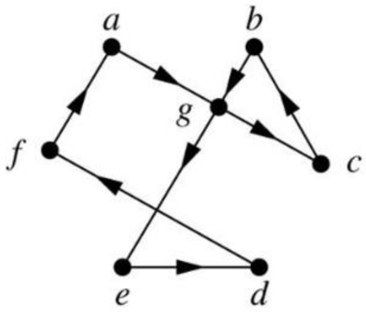
a) 

b) 

c) 

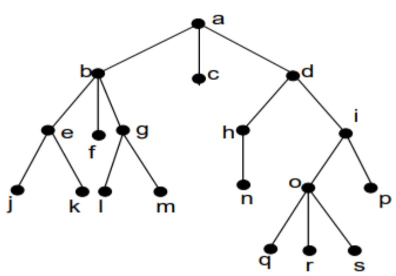
d) 

EXERCISE F



Based on the diagram, determine whether the graph has an Euler Path and Euler Circuit. Hence, construct such path and circuit if they exist or state the reason if they do not exist.

EXERCISE G



Question:

a) Which vertex is the root?

b) Which vertices are internal?

c) Which vertices are leaves?

d) Which vertices are children of 0?

e) Which vertex is the parent of g?

f) Which vertices are siblings of k?

g) Which vertices are ancestors of m?

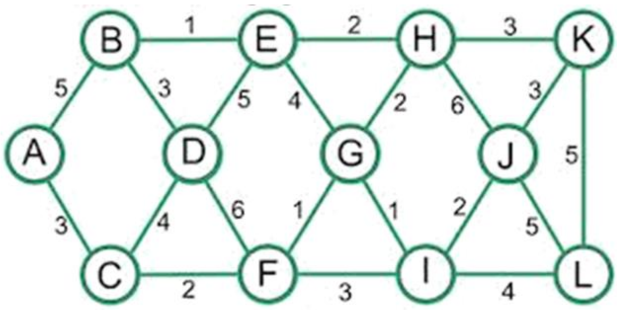
h) Which vertices are descendants of d?

i) What is the level of each vertex of the rooted tree above?

k) Draw a subtree that is rooted at b.

EXERCISE H

1. Based on the following figure,



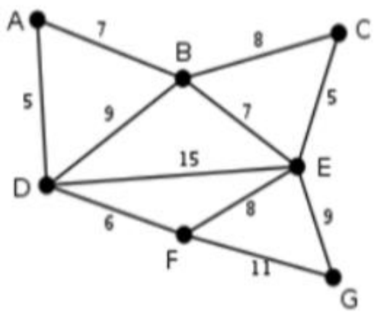
a) Draw 2 possible spanning trees

b) Find a minimum spanning tree by using Kruskal's and Prim's algorithms (compare the findings)

c) Draw the minimum spanning tree.

EXERCISE H

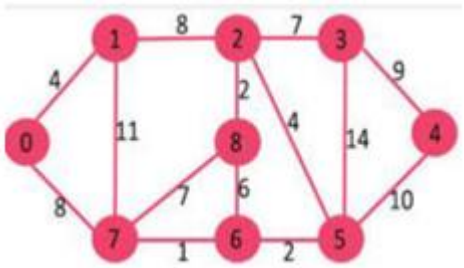
2. By referring to the following weighted graph,



a. Draw the minimum spanning tree by using Kruskal algorithm.

b. Calculate the shortest path.

3. Based on the following graph:



i. Draw the minimum spanning tree by using Prim's algorithm.

ii. Calculate the shortest path.

ADDITIONAL QUESTIONS

1. Determine whether the given graph in Figure 3 and Figure 4 has an Euler circuit or not. Construct such a circuit if it exists.

a)

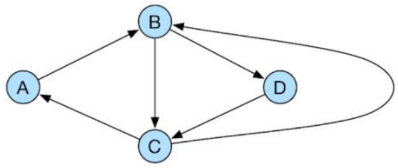


Figure 3

b)

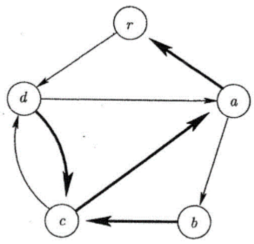


Figure 4

2. For the directed graph below,
   a) Give a Hamiltonian path if it exists. If not, state why.
   b) Give a Hamiltonian circuit if it exists. If not, state why.
   c) List the in- degrees and out degrees for each vertex in a table.

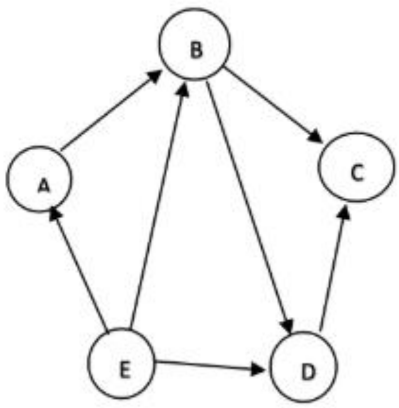


Figure 5

3. Answer these questions about the rooted tree as illustrated in Figure 7.

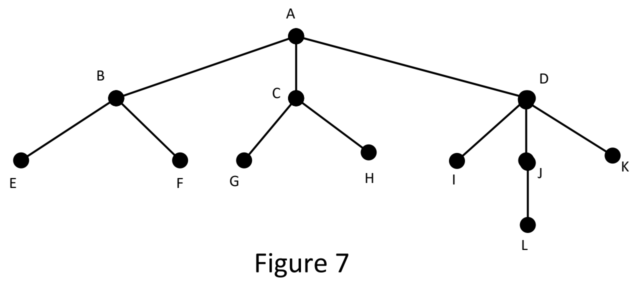


Figure 7

a) Which vertex is the root?

b) Which vertices are leaves?

c) Which vertices are children of C?

d) Which vertices are siblings of J?

4. Find two (2) possible spanning trees for the following weighted graph in Figure 8.

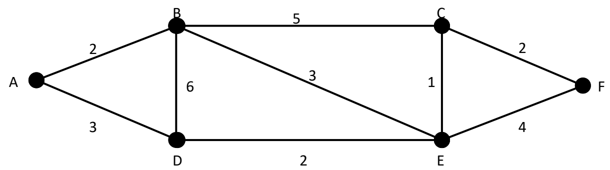


Figure 8

5. The Roads represented by the graph in Figure 9 are all unpaved. The lengths of the roads between two towns are represented by edge weights. Using Prim's algorithm, which road should be paved so that there is a path with a minimum road length? (Begin at Alor Setar and end at Jitra)

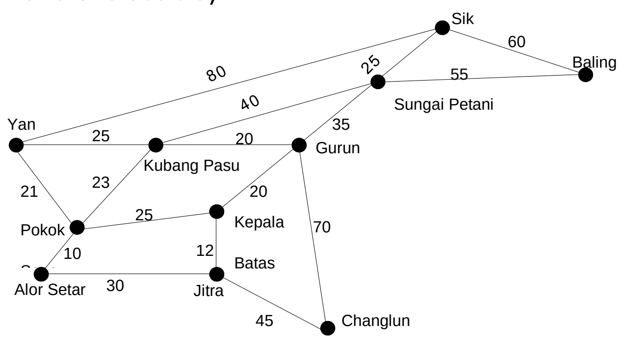


Figure 9

6. Consider the following graphs in Figure 11

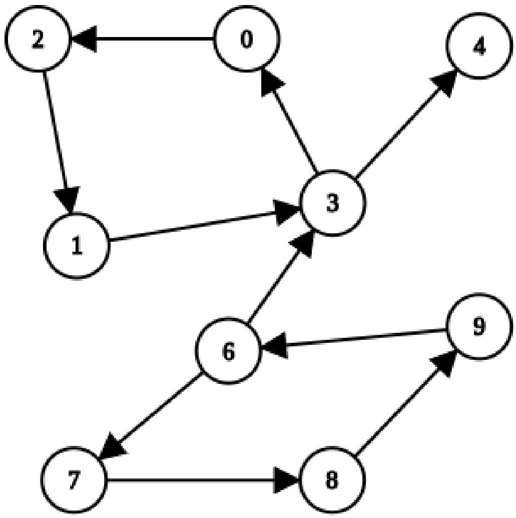


Figure 10

a) Determine whether the graph above has the Euler circuit or not. Explain your answer. Construct the circuit if it exists.

b) Determine whether the graph above has the Hamilton path or not. Explain your answer. Construct the path if it exists.

7. Give the correct answer for each of the following statement.

a) A graph with a number is assigned to each edge.

b) A path that begins and ends at the same vertex.

c) An undirected graph with multiple edge and loop.

d) A vertex with degree zero

8. State the suitable graph terminology for each of the following statements.
    a) A graph with numbers on the edges

    b) A graph without any loop and parallel edges

    c) A Graph with loop and parallel edges

    d) A vertex of degree zero

    e) A vertex with degree one

9.

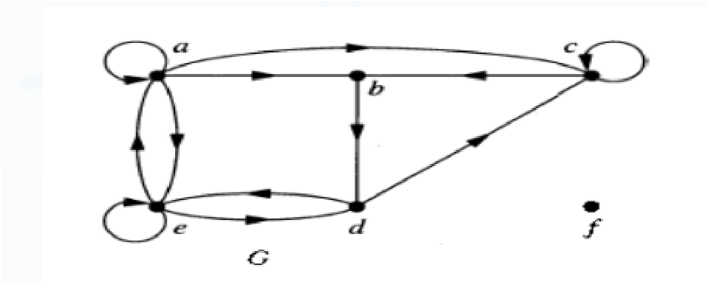


Figure 11

Based on Figure 11
    a) Identify isolated vertex

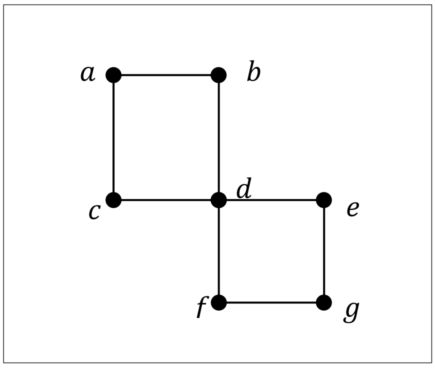
    b) Identify pendant edges

    c) Identify the loops.

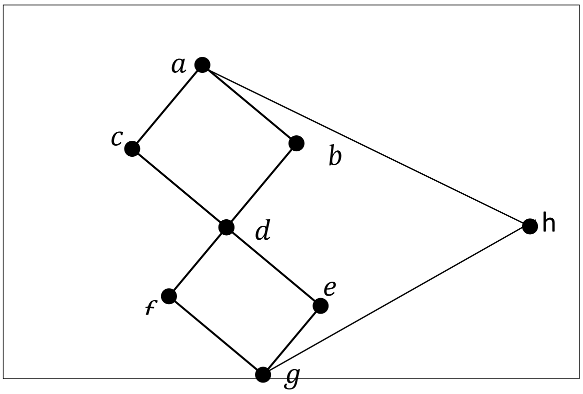
    d) How is the size of the graph is being determined and what is the size of the graph?

    e) State the degree of vertices a,b,c and f.

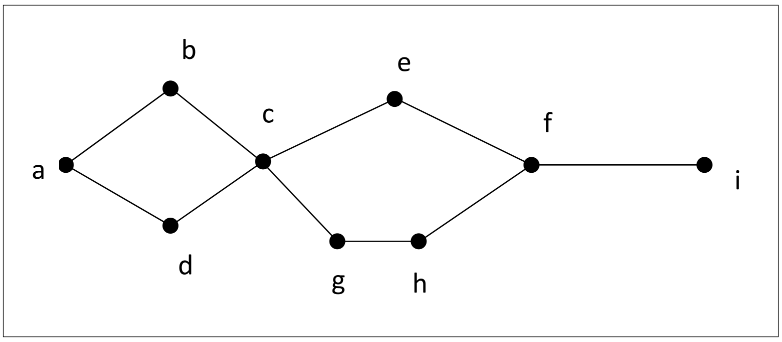
10.



Graph X



Graph Y



Graph Z

Consider the graphs above:

i. Determine which graph has the Euler circuit and then determine which graph has the Euler Path. Explain why the graph has Euler path or Euler circuits.

ii. Construct the circuit and path in (i) if it exists.

11. State the terminology for the following definitions.

a) A path that starts and ends at the same vertex.

b) The edge linking vertex to itself.

c) The path with no repeated vertices.

d) A vertex with degree 1.

e) A path contains at most two vertices of odd degree.

12. Consider the following graphs in Figure 14

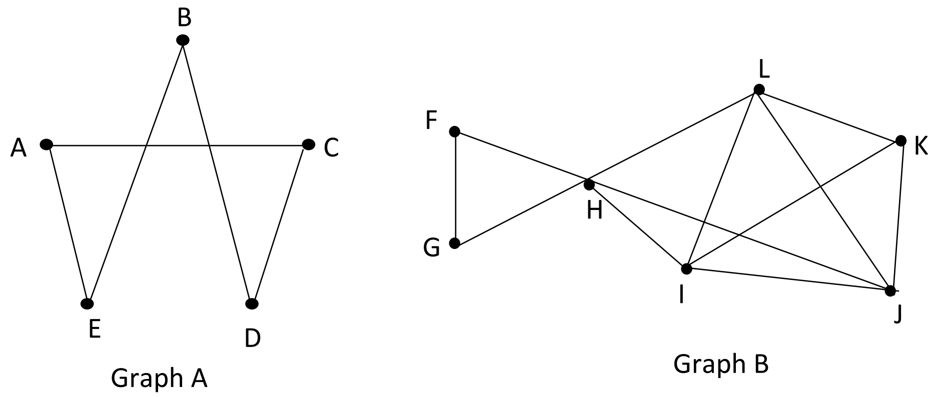


Figure 14

a) Determine which graphs have a Euler circuit. Explain your reason. Construct two different circuits if it exists.

b) Determine which graphs have a Hamilton path. Explain your reason. Construct two different paths if it exists.

CHAPTER 3

Basic Logic

Meaning of proposition

A **proposition** (or **statement**) is a sentence that is either **True** or **False**.

Example of proposition

- 10÷2=4
- 5 is an even number.
- Today is Wednesday

Example of non-proposition

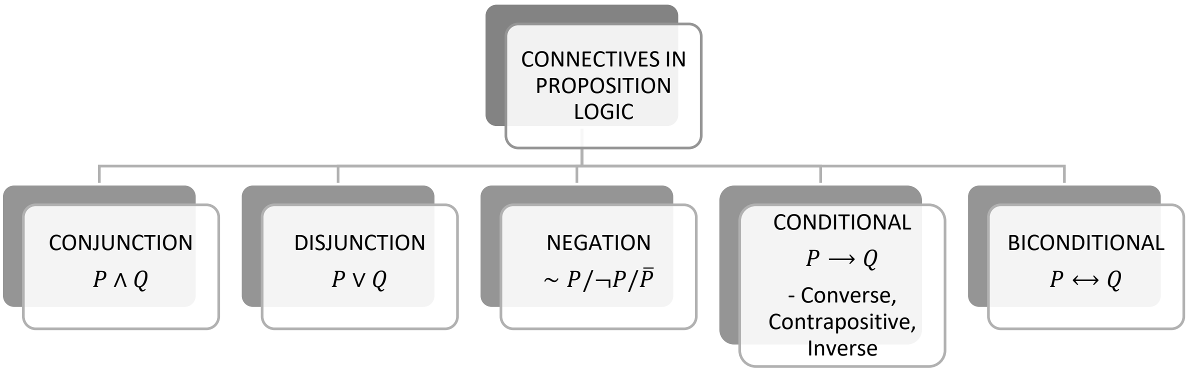
- Where do you live?
- Please answer the
- question correctly.
- x < 10

TWO PROPOSITIONS

p	***q***
T	T
T	F
F	T
F	F

THREE PROPOSITIONS

p	***q***	***r***
T	**T**	**T**
T	**T**	**F**
T	**F**	**T**
T	**F**	**F**
F	**T**	**T**
F	**T**	**F**
F	**F**	**T**
F	**F**	**F**



Tautology

- A proposition $P(p, q, r, \dots \dots)$ is a ***tautology*** if it contains **only T in the last column** of its truth table.

Contigency

- A proposition $P(p, q, r, \dots \dots)$ is a ***contingency*** if it contains both **T** and **F** in the last column of its truth table.

Contradiction

- A proposition $P(p, q, r, \dots \dots)$ is a ***contradiction*** if it contains **only F in the last column** of its truth table.

Type of Quantifiers

Universal quantifier ($\forall \boldsymbol{x}$)
$\forall$x P(x) = for every x, P(x).

Existential quantifier ($\exists \boldsymbol{x}$)
$\exists$x P(x) = There exist an x in P(x).

SOME EXAMPLE OF QUANTIFIERS

Let the universe be the set of airplanes and let $F(x, y)$ denote "**x** flies faster than **y**". Write each proposition in words.

a) $\forall x \forall y F(x, y)$ ⟹ "Every airplane is faster than every airplane"

b) $\forall x \exists y F(x, y)$ ⟹ "Every airplane is faster than some airplane"

c) $\exists x \forall y F(x, y)$ ⟹ "Some airplane is faster than every airplane"

d) $\exists x \exists y F(x, y)$ ⟹ "Some airplane is faster than some airplane"

EXERCISE A

1. Which of these sentences are proportions? What are the truth values of those that are propositions?

a) Kuala Lumpur is the capital of Malaysia.

b) 8+2=10

c) $-48 < -47$

d) Do you want to go to a cinema?

e) Answer this question.

f) $\mathrm{x} + 2 = 18$

g) Today is Monday.

h) Move this table to the other room

EXERCISE B

1. Determine whether the statements are true (T) or false (F).

a) 3+2=5 and 4+4=8

b) Changlun is in Perlis and Alor Setar is in Kedah.

c) $-48 < -47$ and 25+3=38

d) Duck has 4 legs and cat has wings.

e) $4\mathrm{x} + 3\mathrm{x} = 5\mathrm{x}$ and $\frac{5}{4} + \frac{3}{7} = \frac{47}{28}$

EXERCISE 3C

1. Determine whether the statements are true (T) or false (F).

   a) 3+2=5 or 4+4=8

   b) Changlun is in Perlis or Alor Setar is in Kedah.

   c) $-48 < -47$ or 25+3=38

   d) Duck has 4 legs or cat has wings.

   e) $4x + 3x = 5x$ or $\frac{5}{4} + \frac{3}{7} = \frac{47}{28}$

EXERCISE D

1. What is the negation of each of these propositions?

a) Today is Tuesday.

c) China is in Asia

d) $2 + 1 = 3$

e) All kittens are cute.

f) No prime number is even.

g) Some cookies are sweet.

h) Every lawyer uses logic.

i) No bullfrog has lovely eyes.

EXERCISE E

1. Let p be "It is cold" and q be "It is raining". Give a simple sentence which describes each of the following statements:
   a) $p \rightarrow q$
   b) $q \rightarrow \neg p$
   c) $\neg q \rightarrow \neg p$
2. State the converse, contrapositive, and inverse of the following conditional statements "I come to class whenever there is going to be quiz"

EXERCISE F

1. Let p be "It is cold" and q be "It is raining". Give a simple sentence which describes each of the following statements:
   a) $p \leftrightarrow q$
   b) $q \leftrightarrow \neg p$

EXERCISE G/1

1. Which of these sentences are propositions? State the truth value of those that are propositions?
   a) If it snows, then the schools are closed.
   b) $x + 2$ is positive.

c) Take the umbrella with you.

d) No prime number is even.

e) A triangle is not a polygon. (**polygon is a closed path*)

EXERCISE G/2

Let p and q be the propositions

p: Andy is going to Brunei

q: Andy is having a holiday.

Express each of these propositions as an English sentence.

a) $\neg p$

b) $q \vee \neg p$

c) $\neg p \wedge \neg q$

d) $p \leftrightarrow q$

EXERCISE G/3

Represent the sentences below as propositional expressions:

a) Tom is a math major but not computer science major.

b) You can either stay at the hotel and watch TV or you can go to the museum

c) If it is below freezing, it is also snowing.

EXERCISE G/4

Determine whether each of these statements is true or false.

(a) If 1+1=2, then 2+2=5

(b) If monkeys can fly, then 1+1=3

(c) 2+2=4 if and only if 1+1=2

(d) 0>1 if and only if 2>1

EXERCISE H

1. Construct the truth table of these compound propositions.

a) $\neg p \wedge q$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\neg \boldsymbol{p}$	$\neg \boldsymbol{p} \wedge \boldsymbol{q}$

b) $(\sim p \vee q) \rightarrow \sim q$

p	q				

c) $p \wedge (\neg q \vee r)$

p	q	r			

d) $\neg p \leftrightarrow (q \vee r)$

EXERCISE I

1. Construct the truth table for each of the following:

(a) $\neg(q \vee p)$

(b) $\neg(p \wedge q) \rightarrow \neg q$

(c) $\neg q \wedge (p \vee r)$

(d) $(q \, V \, r) \leftrightarrow \neg p$

EXERCISE J

1. Use the truth table to determine whether the statement $((\boldsymbol{p} \rightarrow \boldsymbol{q}) \wedge \boldsymbol{p}\,) \rightarrow \boldsymbol{q}$ is a tautology, contradiction or contingency.

2. Use a truth table to show that the proposition $\boldsymbol{p} \vee (\boldsymbol{q} \vee \sim \boldsymbol{p})$ is always true (T).

3. Determine whether the proposition is tautology or not**,** $(\boldsymbol{p} \rightarrow \boldsymbol{q}) \wedge (\boldsymbol{q} \rightarrow \boldsymbol{p}) \leftrightarrow (\boldsymbol{p} \rightarrow \neg \boldsymbol{q})$**.**

EXERCISE K

1. Let P(x) be the statement “the word x contains the letter a”. What are these truth values?

    a) P (orange)

    b) P (lemon)

    c) P (false)

2. Let P(x) be the statement “x is the states in Malaysia that starts with the letter P”. Find the truth set of P(x), where the domain is all the state in Malaysia.

3. Let P(x) be the statement $x = x^2$. If the domain consists of the integers, what are the truth values?

    a) P (0)

    b) P (1)

    c) P (2)

    d) P (-1)

EXERCISE L

1. Let $Q(x, y)$ denote the statement "$x = y + 3$". What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

2. Let $A(c, n)$ denote the statement "Computer c is connected to network n," where c is a variable representing a computer and n is a variable representing a network? Suppose that the computer $MATH1$ is connected to network $CAMPUS2$, but not to network $CAMPUS1$. What are the truth values of $A(MATH1,\ CAMPUS1)$ and $A(MATH1,\ CAMPUS2)$?

3. Let Q (x, y, z) denote the statement "$x^2 + y^2 = z^2$". What is the truth value of Q (3, 4, 5)? What is the truth value of Q (2, 2, 3)?

EXERCISE M

1. Translate the specifications into English sentences where P(x) be the predicate "x must take a discrete mathematics course" and let Q(x) be the predicate "x is a computer science student". The universe of discourse for both P(x) and Q(x) is all students.

   a) $\forall x\ (Q(x) \land P(x))$

   b) $\exists x\ (P(x) \rightarrow Q(x))$

   c) $\neg\ (\forall x\ Q(x) \rightarrow P(x))$

EXERCISE N

1. Let S(x,y) be the predicate "x is expensive than y" and let the universe of discourse be the set of cars. Express the following in sentences:

   a) $\exists x\ \exists y\ S(x,y)$

   b) $\exists x\ \neg S(x,\text{Mercedes})$

   c) $\neg\forall x\exists y\ S(x,y)$

2. Let P(x): 'x likes sport'.
   Let Q(x): 'x can speak English'.

   The domain for x is the set of all lecturers in Polytechnic. Translate symbolically the following expressions:

   a) Some lecturers in Polytechnic like sport and can speak English.

   b) Every lecturer in Polytechnic like sport if they cannot speak English.

ADDITIONAL EXERCISES

1. Which of these sentences are propositions? State the truth values of those that are propositions.
   a) Sunday is the day after Saturday.

   b) I love teddy bear!

   c) Is 2 is a positive number?

   d) $2n + 3 > 6$, let $n = 2$

2. Let p, q, and r be the following statements.

   p: You study hard;

   q: You will get a good job;

   r: You are happy.

   Express each of these propositions into Logical Connectives.

   a. If you study hard, then you will get a good job.

   b. You are happy if and only if you study hard and you will get a good job.

   c. If you do not study hard, you will not get a good job or not be happy.

3. Let P(x) be the statements “x can speak Russian” and let Q(x) be the statement “x knows the computer language C++”. Express each of these sentences in terms of P(x), Q(x), quantifiers and logical connectives. The domain for quantifiers consists of all students at your school.

   a) All students at your school can speak Russian and knows C++.

   b) There is a student at your school who can speak Russian but who doesn’t know C++.

4. What rule of inferences is used in each of these arguments?

   a) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

   b) If it snows today, the children are happy. The children are not happy today. Therefore, it did not snow today.

   c) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

5. Assuming p is “Amir will go to Japan” and q is “Amir intends to buy souvenirs.”. Express each of the following statements in English sentence form.

   a) $q \vee \sim p$

   b) $\sim p \wedge \sim q$

   c) Show that $(p \vee q) \vee\sim p$ is a tautology.

6. Classify whether the following sentences are proposition or non-propositions.

   a) Clean up your room.

   b) $\mathrm{x} + 2 = 2\mathrm{x}, \text{when } \mathrm{x} = 2$

   c) The product of 3 and 4 is 11.

7. Prepare a truth table for the proposition $\sim((p \wedge q) \wedge r)$ and show whether the proposition is tautology, contradiction or contingency.

8. Let $M(x, y)$ is a predicate "x has sent an email message to y" where the universe of discourse consists of all students in your class. Use quantifiers to express each of the following statements.

   a) Alia has never sent an email message to Nurin.

   b) Every student in your class has sent an email message to Sarah.

c) There is a student in your class who sent an email message to everyone in your class.

d) Every student in the class has sent an email message to some students in the class.

9. Assume $P(x)$ is the statement of “x is perfect” and $F(x)$ is “x is your friend”, whereby the domain of x consists of all people. Translate each of these statements into logical expressions using predicates, quantifiers and logical connectives

   a) All people are not perfect.

   b) At least one of your friends is perfect.

   c) Not everybody is your friend or someone is not perfect.

CHAPTER 4

Boolean Algebra

BOOLEAN ALGEBRA

- **BOOLEAN SUM**
- **BOOLEAN PRODUCT**
- **COMPLEMENTATION**

COMPLEMENTATION

X	**X'**
0	1
1	0

BOOLEAN SUM

X	Y	F=X+Y
0	0	0
0	1	1
1	0	1
1	1	1

BOOLEAN PRODUCT

X	Y	F=X•Y
0	0	0
0	1	0
1	0	0
1	1	1

Example 1:

Find the value of
$1.0 + (\overline{0 + 1})$

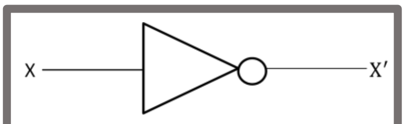
Solution:
$1.0 + (\overline{0 + 1})$
$= 0 + (\overline{1})$
$= 0 + 0$
$= 0$

IDENTITIES OF BOOLEAN ALGEBRA

IDENTITY	NAME
$\overline{\overline{x}} = x$	**Law of the double complement**
$x + x = x$ $x.x = x$	**Idempotent laws**
$x + 0 = x$ $x.1 = x$	**Identity laws**
$x + 1 = 1$ $x.0 = 0$	**Domination laws**
$x + y = y + x$ $xy = yx$	**Commutative laws**
$x + (y + z) = (x + y) + z$ $x(yz) = (xy)z$	**Associative laws**
$x + yz = (x + y)(x + z)$ $x(y + z) = xy + xz$	**Distributive laws**
$\overline{(xy)} = \overline{x} + \overline{y}$ $\overline{(x + y)} = \overline{x}\overline{y}$	**De Morgan's laws**
$x + xy = x$ $x(x + y) = x$	**Absorption laws**
$x + \overline{x} = 1$	**Unit property**
$x\overline{x} = 0$	**Zero property**

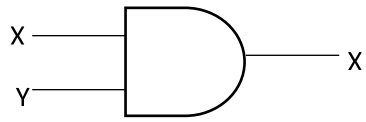
LOGIC GATES

1- The Inverter



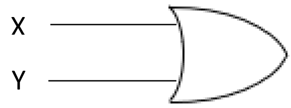
Performs the Boolean **NOT** operation.

1- The AND Gate



Performs the Boolean **AND** operation

1- The OR Gate

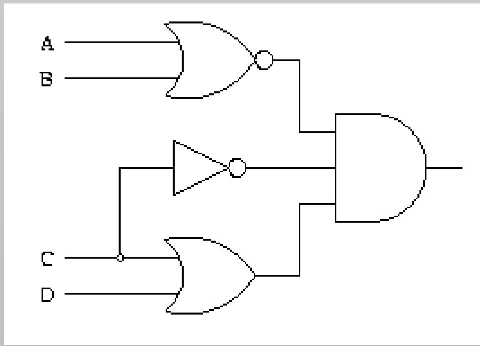


Performs the Boolean **AND** operation

Example 2 :

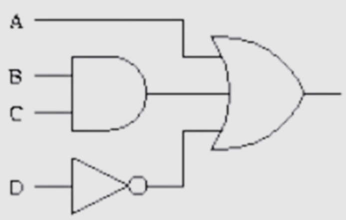
DRAW A LOGIC CIRCUIT CORRESPONDING TO THE BOOLEAN EXPRESSION

$(\overline{A + B})(C + D)\bar{C}$

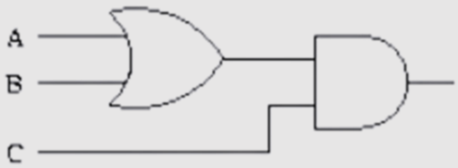


Example 3:

EXPRESS THE OUTPUT FROM THE GIVEN CIRCUITS



$Y = A + BC + \overline{D}$



$Y = (A + B)C$

Example 4:

MINIMIZE BOOLEAN EXPRESSION USING KARNAUGH MAP

Minimize the following Boolean expression by using K-map.

$$F(x, y, z) = xyz + xy\bar{z} + x\bar{y}z$$

	$\boldsymbol{yz}$	$\boldsymbol{y\bar{z}}$	$\boldsymbol{\bar{y}\bar{z}}$	$\boldsymbol{\bar{y}z}$
$\boldsymbol{x}$	1	1	0	1
$\boldsymbol{\bar{x}}$	0	0	0	0

$$= \boldsymbol{xy + xz}$$

EXERCISE A

1. Find the values of these expressions.

   a) $\mathbf{1 \cdot \overline{0}}$

   b) $\mathbf{1 + \overline{1}}$

   c) $\mathbf{\overline{0} \cdot 0}$

   d) $\mathbf{(\overline{1 + 0})}$

EXERCISE B

Simplify the following Boolean expression:

a) $\mathrm{C} + \overline{\mathrm{BC}}$

b) $\overline{\mathrm{AB}}(\overline{\mathrm{A}} + \mathrm{B})(\overline{\mathrm{B}} + \mathrm{B})$

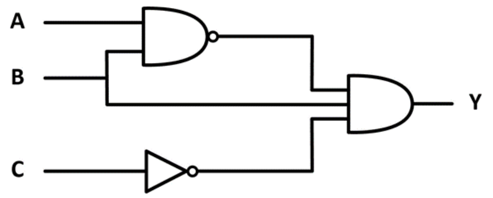
c) $(x + y)(xz + xz') + xy + y$

d) $\bar{x}(x + y) + (y + xx)(x + \bar{y})$

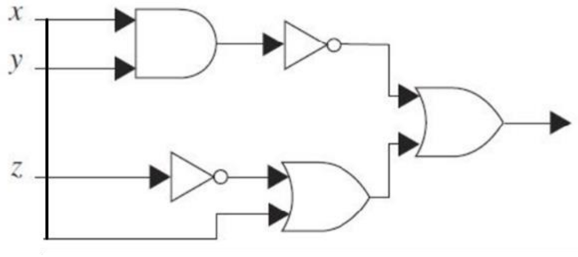
EXERCISE C

1. Find the output of the given circuits:

a)



b)



2. Draw a logic circuit corresponding to the Boolean expression:

   a) $\mathrm{Y} = (\mathrm{A}+\mathrm{B})\mathrm{C}$

   b) $\mathrm{Y} = A + BC + \overline{D}$

   c) $\mathrm{Y} = \overline{\mathrm{A} + \mathrm{BC}} + \mathrm{B}$

   d) $\mathrm{Y} = \overline{\mathrm{A}'\mathrm{B}} + \overline{\mathrm{A} + \mathrm{C}}$

EXERCISE D

1. Find the truth table T for the equivalent Boolean expression:

   $F(A, B, C) = ABC' + BC' + A'B$

EXERCISE E

1. Here is a truth table for a specific three input logic circuit.

   Draw a K Map according to the values found in the truth table.

A	B	C	OUTPUT
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

2. Use Karnaugh maps to find the minimal form for each expression.

   a) $xy + xy'$

   b) $xy + x'y + x'y'$

   c) $xy' + x'y'$

   d) $xyz' + xy'z + xy'z' + x'yz + x'yz' + x'y'z$

   e) $xyz + xyz' + x'yz + x'y'z$

f) $xyz + xyz' + xy'z + xy'z' + x'y'z$

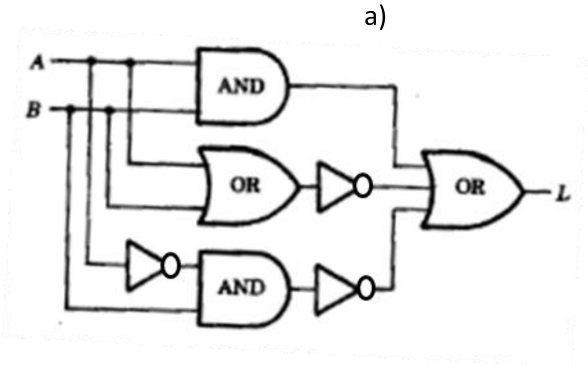
3. Design a minimal AND-OR circuit which yields the following truth table:

$$T = [A = 00001111,\ B = 00110011,$$
$$C = 01010101,\ L = 10101001]$$

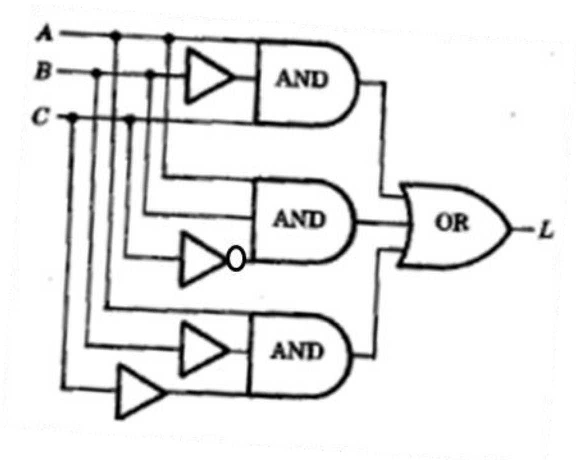
EXERCISE E

4. Redesign the following circuit so that it becomes a minimal AND-OR circuit

a)



b)



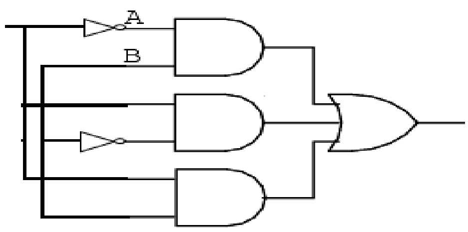
ADDITIONAL EXERCISE

1. Simplify the following expression using a Karnaugh map:

 i. $F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

 ii. $F(x,y,z) = \bar{x}yz + \bar{x}\bar{y}z + xy\bar{z} + \bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + xyz$

2. Consider the following circuit. Minimize the circuit using Karnaugh map.



3. Simplify $F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$ using a Karnaugh map and draw the simplified circuit.

4. Use a K Map to simplify the following expressions:

   i. $\mathrm{x} = \overline{\mathrm{B}}\mathrm{C} + \overline{\mathrm{A}}\overline{\mathrm{B}}\overline{\mathrm{C}} + \mathrm{A}\overline{\mathrm{B}}\overline{\mathrm{C}} + \mathrm{AB}\overline{\mathrm{C}}$

   ii. $\mathrm{y} = \mathrm{ABC} + \overline{\mathrm{A}}\mathrm{B}\overline{\mathrm{C}} + \overline{\mathrm{C}} + \overline{\mathrm{A}}\mathrm{BC}.$

   iii. $\mathrm{z} = \overline{\mathrm{A}} + \mathrm{AB}\overline{\mathrm{C}}$

2. Use a K Map to simplify the following expressions:

    i. $\mathrm{x} = \overline{\mathrm{A}}\overline{\mathrm{B}}\overline{\mathrm{C}} + \overline{\mathrm{A}}\overline{\mathrm{B}}\mathrm{C} + \overline{\mathrm{A}}\mathrm{BC} + \overline{\mathrm{A}}\mathrm{B}\overline{\mathrm{C}} + \mathrm{A}\overline{\mathrm{B}}\overline{\mathrm{C}} + \mathrm{AB}\overline{\mathrm{C}}$

    ii. $\mathrm{F} = \overline{\mathrm{A}}\overline{\mathrm{B}}\overline{\mathrm{C}} + \overline{\mathrm{A}}\overline{\mathrm{B}}\mathrm{C} + \overline{\mathrm{A}}\mathrm{BC} + \overline{\mathrm{A}}\mathrm{B}\overline{\mathrm{C}} + \mathrm{ABC} + \mathrm{AB}\overline{\mathrm{C}}$

    iii. $\mathrm{G} = \overline{\mathrm{A}}B + \mathrm{B}\overline{\mathrm{C}} + BC + A\bar{B}\bar{C}$

DBM 20153

PAST YEARS QUESTION

SULIT

KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

BAHAGIAN PEPERIKSAAN DAN PENILAIAN
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
KEMENTERIAN PENDIDIKAN TINGGI

JABATAN MATEMATIK, SAINS DAN KOMPUTER

PEPERIKSAAN AKHIR

SESI I : 2024/2025

DBM20153: DISCRETE MATHEMATICS

TARIKH : 02 DISEMBER 2024
MASA : 2.30 PTG – 4.30 PTG (2 JAM)

Kertas ini mengandungi **SEPULUH (10)** halaman bercetak.

Struktur (4 soalan)

Dokumen sokongan yang disertakan : Tiada

JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIARAHKAN

(CLO yang tertera hanya sebagai rujukan)

SULIT

INSTRUCTION:

This section consists of **FOUR (4)** structured questions. Answer **ALL** questions.

ARAHAN:

*Bahagian ini mengandungi **EMPAT (4)** soalan berstruktur. Jawab **SEMUA** soalan.*

CLO1

QUESTION 1

SOALAN 1

(a) Given that:

Set $\xi = \{x: 1 \leq x \leq 30, x \text{ is an integer}\}$

Set A = $\{x: x$ is an even number$\}$

Set B = $\{x: x$ is a multiple of 4$\}$

Set C = $\{x: x$ is a multiple of 3$\}$

Diberi :

Set $\xi = \{x: 1 \leq x \leq 30, x \text{ adalah integer}\}$

Set A = $\{x$: *x adalah nombor genap*$\}$

Set B = $\{x$: *x adalah nombor gandaan 4*$\}$

Set C = $\{x$: *x adalah nombor gandaan 3*$\}$

Express the following:

Nyatakan yang berikut:

i. Elements of $(A \cap B)' \cap C'$ [7 marks]

[*7 markah*]

ii. Elements of $[(A \cup C) \cap B]'$ [3 marks]

[*3 markah*]

iii. $n(A' \cap B' \cap C)$ [5 marks]

[*5 markah*]

(b) Given a relation R = {(1,1), (1,3), (2,1), (2,2), (2,4), (3,3), (3,4), (4,2), (4,3)} on set A = {1,2, 3, 4}.
Diberi hubungan R {(1,1), (1,3), (2,1), (2,2), (2,4), (3,3), (3,4), (4,2), (4,3)} *ke atas set A = {1,2, 3, 4}.*

i. Draw a directed graph of the relation
Lukiskan graf berarah bagi hubungan tersebut

[5 marks]
[*5 markah*]

ii. Identify whether the relation is reflexive, symmetric or transitive. Hence determine whether the relation is an equivalence relation. State the reason if the relation is not equivalence relation.
Kenalpasti sama ada hubungan ini refleksif, simetri atau transitif. Kemudian tentukan sama ada ia mempunyai hubungan setara. Nyatakan sebab jika hubungan bukan hubungan setara.

[5 marks]
[*5 markah*]

CLO 1

QUESTION 2

SOALAN 2

(a) Based on Diagram 2(a) :

Berdasarkan Rajah 2(a):

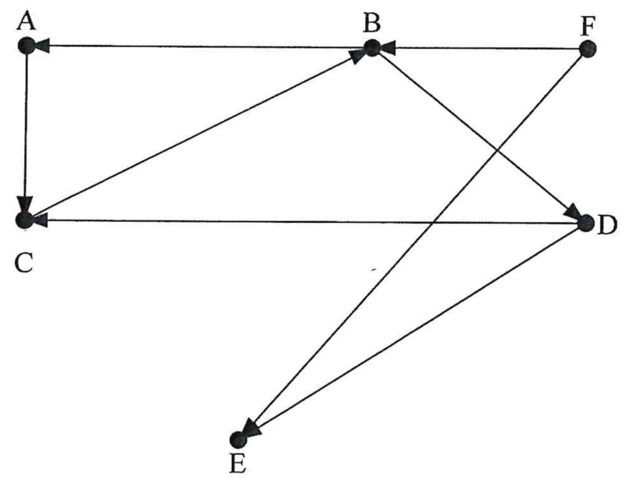


Diagram 2(a) / *Rajah 2(a):*

i. Identify the in-degree and out-degree for vertex D, E and F.

Tentukan darjah-masuk dan darjah-keluar bagi vertek D, E dan F.

[3 marks]

[*3 markah*]

ii. Is F – B – A – C – B – D – C a path? State your reason.

Adakah F – B – A – C – B – D – C suatu laluan? Nyatakan alasan anda.

[2 marks]

[*2 markah*]

(b) Based on Diagram 2(b):
Berdasarkan pada Rajah 2(b):

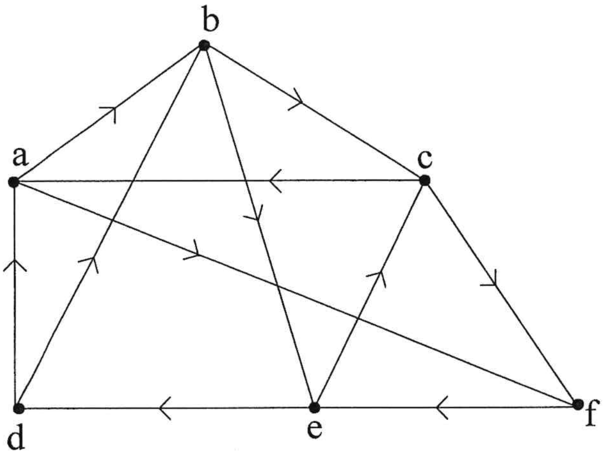


Diagram 2(b) / *Rajah 2(b)*

i. Does Hamilton circuit exist? If yes, construct a Hamilton circuit.
Adakah litar Hamilton wujud? jika ya,, bina litar Hamilton.

[4 marks]
[*4 markah*]

ii. Determine whether the graph has an Euler Path and Euler Circuit. Hence, construct such path and circuit if they exist or state the reason if they do not exist.
Tentukan sama ada graf mempunyai Laluan Euler dan Litar Euler. Kemudian, bina laluan dan litar tersebut jika ia wujud atau berikan sebab jika ia tidak wujud.

[6 marks]
[*6 markah*]

(c) Based on Diagram 2(c):
Berdasarkan pada Rajah 2(c):

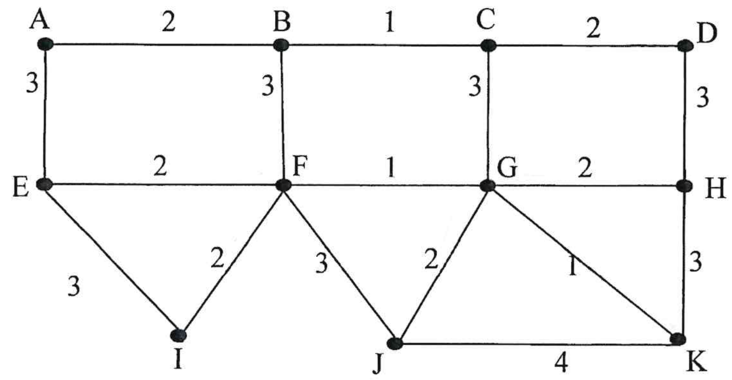


Diagram 2(c) / *Rajah 2(c)*

i. Construct a minimum spanning tree for the weighted graph by using Kruskal's algorithm.
Bina pohon rentangan minimum bagi graf pemberat dengan menggunakan algoritma Kruskal.

[5 marks]
[*5 markah*]

ii. Calculate the shortest path for the weighted graph by using Prim's algorithm using vertex K as the starting point.
Kirakan laluan terpendek bagi graf pemberat dengan menggunakan algoritma Prim dan guna bucu K sebagai titik permulaan.

[5marks]
[*5 markah*]

CLO 1

QUESTION 3

SOALAN 3

Given the statements:

P : Zelia has an email account

Q: Zelia can log in to Instagram

R : Zelia can be an instafamous

S : Zelia can get a lot of money

Diberi penyataan – penyataan :

P : Zelia mempunyai akaun email

Q : Zelia boleh log masuk ke Instagram

R : Zelia boleh menjadi instafamous

S : Zelia boleh mendapat banyak duit

(a) Express the following compound propositions in English sentence:
Ungkapkan proposisi majmuk berikut dalam bahasa Inggeris.

i. $(P \wedge Q) \rightarrow R$ [2 marks]
[2 *markah*]

ii. $(\sim R \rightarrow \sim S) \leftrightarrow (\sim P \vee \sim Q)$ [3 marks]
[3 *markah*]

(b) Express the following compound propositions into symbolic form:
Berdasarkan pernyataan – pernyataan dalam Soalan 3(a), ungkapkan proposisi majmuk berikut dalam bentuk simbol:

i. Zelia can get a lot of money if and only if she can log in to Instagram and be an instafamous.
Zelia boleh mendapat banyak duit jika dan hanya jika dia boleh log masuk ke Instagram dan menjadi instafamous.

[2 marks]
[2 *markah*]

ii. If Zelia has an email account and can log in to Instagram then she can be an instafamous but she cannot get a lot of money.
Jika Zelia mempunyai akaun email dan boleh log masuk ke Instagram maka dia boleh menjadi instafamous tetapi dia tidak boleh mendapat banyak duit.

[3 marks]
[*3 markah*]

(c) Construct truth table to show that whether the following proposition:
Bina jadual kebenaran untuk menunjukkan sama ada proposisi berikut:

i. $(\sim P \rightarrow Q) \vee P$ is a tautology.
$(\sim P \rightarrow Q) \vee P$ *adalah tautologi.*

[4 marks]
[*4 markah*]

ii. $(P \wedge \sim Q) \leftrightarrow P$ is a contradiction.
$(P \wedge \sim Q) \leftrightarrow P$ *adalah percanggahan.*

[4 marks]
[*4 markah*]

iii. $(\sim P \rightarrow Q) \vee (\sim R \rightarrow P)$ is a contingency.
$(\sim P \rightarrow Q) \vee (\sim R \rightarrow P)$ *adalah kontingensi.*

[7 marks]
[*7 markah*]

CLO1

QUESTION 4

SOALAN 4

(a) Write the Boolean expression for each output in the following circuit.

Tulis ungkapan Boolean bagi setiap output dalam rajah litar berikut.

i.

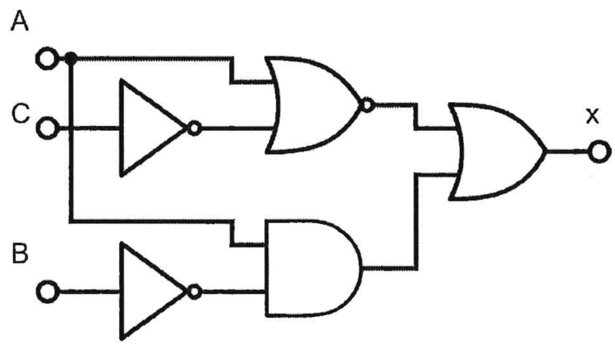


Diagram 4(a)i: Logic circuit

Rajah 4(a)i : Litar logik

[5 marks]

[*5 markah*]

ii.

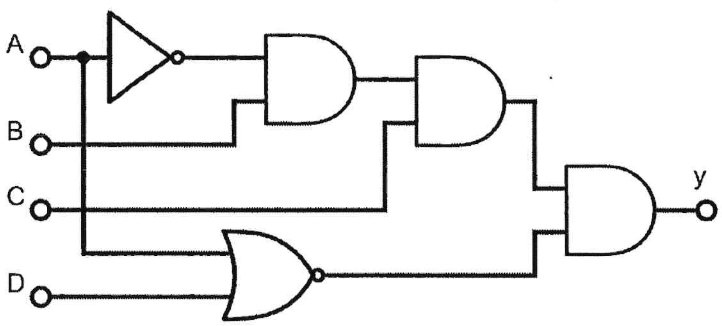


Diagram 4(a)ii: Logic circuit

Rajah 4(a)ii : Litar logik

[5 marks]

[*5 markah*]

CLO1

(b) Apply Karnaugh map to solve the following Boolean expressions:
Gunakan peta Karnaugh untuk selesaikan ungkapan Boolean berikut:

i. $x = AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C$

[5 marks]
[*5 markah*]

ii. $x = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + AB\bar{C}$

[5 marks]
[*5 markah*]

iii. $x = \bar{A}B + BC + +A\bar{B}C + A\bar{C}$

[5 marks]
[*5 markah*]

SOALAN TAMAT

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