POLITEKNIK MALAYSIA
SULTAN ABDUL HALIM MU'ADZAM SHAH

DISCRETE MATHEMATICS

Polytechnic edition

WORKBOOK

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MATHEMATICS, SCIENCE, AND COMPUTER DEPARTMENT

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WORKBOOK

NAME : ....

MATRIC NO. : ....

SESSION: ....

DISCLAIMER

Publisher: POLITEKNIK SULTAN ABDUL HALIM MU'ADZAM SHAH (POLIMAS)

First Published : AUGUST 2023
ISBN:

COPYWRITING
Marina binti Mat Isa
Nor Jamilah binti Ishak

Publisher:
POLITEKNIK SULTAN ABDUL HALIM MU'ADZAM SHAH (POLIMAS) 06000 JITRA, KEDAH
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Fax : 04-9174232/ 04-9146323

PREFACE

Alhamdulillah praise to be Allah S.W.T, with His grace and mercy; peace and blessing upon Prophet Muhammad P.B.U.H. his family and companions; this workbook has been completed.

The content of this workbook has been designs to meet the syllabus requirements of polytechnics so that students can make use out of it. The process of completing the workbook smooths along the way since we were experienced in teaching and learning of this course for several years

This workbook caries synopsis notes and exercises from 5 chapter. Every chapters introduces a synopsis of notes and is augmented by details worked examples that lead on to the questions for the exercises attached in the end of the section.

We hope to see this workbook as a catalyst to attracted interest students to go on learning more related advanced material such as Discrete Mathematics and Its Applications course.

CONTENTS

CHAPTER 1

Basic Logic And Proofs

Meaning of proposition

A **proposition** (or **statement**) is a sentence that is either **True** or **False**.

Example of proposition

- $10 \div 2 = 4$
- 5 is an even number.
- Today is Wednesday

Example of non-proposition

- Where do you live?
- Please answer the
- question correctly.
- x < 10

TWO PROPOSITIONS

p	***q***
T	T
T	F
F	T
F	F

THREE PROPOSITIONS

p	***q***	***r***
T	**T**	**T**
T	**T**	**F**
T	**F**	**T**
T	**F**	**F**
F	**T**	**T**
F	**T**	**F**
F	**F**	**T**
F	**F**	**F**

CONNECTIVES IN PROPOSITION LOGIC

- CONJUNCTION $P \wedge Q$
- DISJUNCTION $P \vee Q$
- NEGATION $\sim P / \neg P / \bar{P}$
- CONDITIONAL $P \longrightarrow Q$
- BICONDITIONAL $P \longleftrightarrow$

Logical equivalence	• Two statement forms are called logically equivalent ($\equiv$) if and only if they have same truth value in every possible situation.
Tautology	• A proposition $P(p, q, r, \ldots \ldots)$ is a ***tautology*** if it contains **only T in the last column** of its truth table.
Contigency	• A proposition $P(p, q, r, \ldots \ldots)$ is a ***contingency*** if it contains both **T** and **F** in the last column of its truth table.
Contradiction	• A proposition $P(p, q, r, \ldots \ldots)$ is a ***contradiction*** if it contains **only F in the last column** of its truth table.

Type of Quantifiers

- Universal quantifier ($\forall \boldsymbol{x}$)
  $\forall$x P(x) = for every x, P(x).
- Existential quantifier ($\exists \boldsymbol{x}$)
  $\exists$x P(x) = There exist an x in P(x).

SOME EXAMPLE OF QUANTIFIERS

Let the universe be the set of airplanes and let $F(x,y)$ denote "**x** flies faster than **y**". Write each proposition in words.

a) $\forall x \forall y F(x,y)$ ➡ "Every airplane is faster than every airplane"

b) $\forall x \exists y F(x,y)$ ➡ "Every airplane is faster than some airplane"

c) $\exists x \forall y F(x,y)$ ➡ "Some airplane is faster than every airplane"

d) $\exists x \exists y F(x,y)$ ➡ "Some airplane is faster than some airplane"

VALIDITY OF ARGUMENT

An argument is said to be ***valid*** if $\boldsymbol{Q}$ is true whenever all the premises $\boldsymbol{P_1}$, $\boldsymbol{P_2}$, … , $\boldsymbol{P_n}$ are true.

TEST THE VALIDITY USING TRUTH TABLE.

Example:

Show that the following argument is valid or fallacy.

a)

$$\begin{array}{l} p \to q \\ \underline{p \qquad\quad} \\ \therefore\ q \end{array}$$

Solution:

p	q	$\boldsymbol{p \to q}$	p	q
T	T	T	T	**T**
T	F	F – ignore!	T	–
F	T	T	F – ignore!	–
F	F	T	F – ignore!	–

VALID

RULE OF INFERENCE	TAUTOLOGY	NAME
$p \to q$ $\underline{\quad p \quad}$ $\therefore q$	$[p \land (p \to q)] \to q$	**Modus ponens**
$p \to q$ $\underline{\quad \neg q \quad}$ $\therefore \neg p$	$[\neg q \land (p \to q)] \to \neg p$	**Modus tollens**
$p \to q$ $\underline{\quad q \to r \quad}$ $\therefore p \to r$	$[(p \to q) \land (q \to r)] \to (p \to r)$	**Hypothetical syllogism**
$p \lor q$ $\underline{\quad \neg p \quad}$ $\therefore q$	$[(p \lor q) \land \neg p] \to q$	**Disjunctive syllogism**
$\underline{\quad p \quad}$ $\therefore p \lor q$	$p \to (p \lor q)$	**Addition**

TEST THE VALIDITY USING TABLE RULES OF INFERENCE.

Alice is mathematics major. Therefore, Alice is either mathematics major or a computer science major.

Solution:

Identify the premise:

p: Alice is mathematics major.

q: Alice is a computer science major

Check the given statement:

p (Alice is mathematics major)

$\therefore p \lor q$

(Therefore, Alice is either mathematics major or a computer science major)

Refer to the rules of inference:

Addition

EXERCISE 1A

1. Which of these sentences are proportions? What are the truth values of those that are propositions?

a) Kuala Lumpur is the capital of Malaysia.

b) 8+2=10

c) $-48 < -47$

d) Do you want to go to a cinema?

e) Answer this question.

f) $x + 2 = 18$

g) Today is Monday.

h) Move this table to the other room

EXERCISE 1B

1. Determine whether the statements are true (T) or false (F).

a) 3+2=5 and 4+4=8

b) Changlun is in Perlis and Alor Setar is in Kedah.

c) $-48 < -47$ and 25+3=38

d) Duck has 4 legs and cat has wings.

e) $4x + 3x = 5x$ and $\frac{5}{4} + \frac{3}{7} = \frac{47}{28}$

EXERCISE 1C

1. Determine whether the statements are true (T) or false (F).

a) 3+2=5 or 4+4=8

b) Changlun is in Perlis or Alor Setar is in Kedah.

c) $-48 < -47$ or 25+3=38

d) Duck has 4 legs or cat has wings.

e) $4x + 3x = 5x$ or $\frac{5}{4} + \frac{3}{7} = \frac{47}{28}$

EXERCISE 1D

1. What is the negation of each of these propositions?

a) Today is Tuesday.

c) China is in Asia

d) $2 + 1 = 3$

e) All kittens are cute.

f) No prime number is even.

g) Some cookies are sweet.

h) Every lawyer uses logic.

i) No bullfrog has lovely eyes.

EXERCISE 1E

1. Let p be "It is cold" and q be "It is raining". Give a simple sentence which describes each of the following statements:

a) $p \rightarrow q$

b) $q \rightarrow \neg p$

c) $\neg q \rightarrow \neg p$

EXERCISE 1F

1. Let p be "It is cold" and q be "It is raining". Give a simple sentence which describes each of the following statements:

a) $p \leftrightarrow q$

b) $q \leftrightarrow \neg p$

EXERCISE 1G/1

1. Which of these sentences are propositions? State the truth value of those that are propositions?

a) If it snows, then the schools are closed.

b) $x + 2$ is positive.

c) Take the umbrella with you.

d) No prime number is even.

e) A triangle is not a polygon. (**polygon is a closed path*)

EXERCISE 1G/2

Let p and q be the propositions

p: Andy is going to Brunei

q: Andy is having a holiday.

Express each of these propositions as an English sentence.

a) $\neg p$

b) $q \vee \neg p$

c) $\neg p \wedge \neg q$

d) $p \leftrightarrow q$

EXERCISE 1G/3

Represent the sentences below as propositional expressions:

a) Tom is a math major but not computer science major.

b) You can either stay at the hotel and watch TV or you can go to the museum

c) If it is below freezing, it is also snowing.

EXERCISE 1G/4

1. Determine whether each of these statements is true or false.

(a) If 1+1=2, then 2+2=5

(b) If monkeys can fly, then 1+1=3

(c) 2+2=4 if and only if 1+1=2

(d) 0>1 if and only if 2>1

EXERCISE 1G/5

1. Construct the truth table of these compound propositions.

a) $\neg p \wedge q$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\neg\boldsymbol{p}$	$\neg\boldsymbol{p} \wedge \boldsymbol{q}$

b) $(\sim p \vee q) \rightarrow \sim q$

$\boldsymbol{p}$	$\boldsymbol{q}$				

c) $p \wedge (\neg q \vee r)$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{r}$			

d) $\neg p \leftrightarrow (q \vee r)$

EXERCISE 1H

1. Construct the truth table for each of the following:

(a) $p \wedge \neg q$

(b) $p \, V \neg \, q \rightarrow q$

(c) $p \vee (\neg q \; V \; r)$

(d) $\neg p \leftrightarrow \sim q \, V \, r$

EXERCISE 1I

1. Show that the statements below are logically equivalent or not.

   a) i: $\neg(\boldsymbol{p} \boldsymbol{\Lambda} \boldsymbol{q})$

      ii: $\neg \boldsymbol{p} \boldsymbol{V} \neg \boldsymbol{q}$

b) i: $\neg \boldsymbol{p} \leftrightarrow \boldsymbol{q}$

ii: $\neg \boldsymbol{q} \leftrightarrow \boldsymbol{p}$

2. Use truth tables to show that: $[(\boldsymbol{p} \boldsymbol{V} \boldsymbol{q}) \rightarrow \boldsymbol{r}] \equiv [(\boldsymbol{p} \rightarrow \boldsymbol{r}) \wedge (\boldsymbol{q} \rightarrow \boldsymbol{r})]$

EXERCISE 1J

1. Use the truth table to determine whether the statement $((\boldsymbol{p} \rightarrow \boldsymbol{q}) \wedge \boldsymbol{p}) \rightarrow \boldsymbol{q}$ is a tautology, contradiction or contingency.

2. Use a truth table to show that the proposition $\boldsymbol{p} \vee (\boldsymbol{q} \vee \sim \boldsymbol{p})$ is always true (T).

3. Determine whether the proposition is tautology or not, $(\boldsymbol{p} \rightarrow \boldsymbol{q}) \wedge (\boldsymbol{q} \rightarrow \boldsymbol{p}) \leftrightarrow (\boldsymbol{p} \rightarrow \neg \boldsymbol{q})$.

EXERCISE 1K

1. Let P(x) be the statement "the word x contains the letter a". What are these truth values?
   a) P (orange)
   b) P (lemon)
   c) P (false)

2. Let P(x) be the statement "x is the states in Malaysia that starts with the letter P". Find the truth set of P(x), where the domain is all the state in Malaysia.

3. Let P(x) be the statement $x = x^2$. If the domain consists of the integers, what are the truth values?
   a) P (0)
   b) P (1)
   c) P (2)
   d) P (-1)

EXERCISE 1L

1. Let $Q(x, y)$ denote the statement "$x = y + 3$". What are the truth values of the propositions $Q(1,2)$ and $Q(3,0)$?

2. Let $A(c, n)$ denote the statement "Computer c is connected to network n," where c is a variable representing a computer and n is a variable representing a network. Suppose that the computer $MATH1$ is connected to network $CAMPUS2$, but not to network $CAMPUS1$. What are the truth values of $A(MATH1,\ CAMPUS1)$ and $A(MATH1,\ CAMPUS2)$?

3. Let Q (x, y, z) denote the statement "$x^2 + y^2 = z^2$". What is the truth value of Q (3, 4, 5)? What is the truth value of Q (2, 2, 3)?

EXERCISE 1M

1. Translate the specifications into English sentences where P(x) be the predicate "x must take a discrete mathematics course" and let Q(x) be the predicate "x is a computer science student". The universe of discourse for both P(x) and Q(x) is all students.

   a) $\forall x\ (Q(x) \land P(x))$

   b) $\exists x\ (P(x) \rightarrow Q(x))$

   c) $\neg\ (\forall x\ Q(x) \rightarrow P(x))$

EXERCISE 1M

2. Let S(x,y) be the predicate "x is expensive than y" and let the universe of discourse be the set of cars. Express the following in sentences:

   a) $\exists x \, \exists y \, S(x,y)$

   b) $\exists x \, \neg S(x,\text{Mercedes})$

   c) $\neg \forall x \exists y \, S(x,y)$

3. Let P(x): 'x likes sport'.
   Let Q(x): 'x can speak English'.

   The domain for x is the set of all lecturers in Polytechnic. Translate symbolically the following expressions:

   a) Some lecturers in Polytechnic like sport and can speak English.

   b) Every lecturer in Polytechnic like sport if they cannot speak English.

EXERCISE 1N

1. Show that the following argument is valid or fallacy.

a) $p \rightarrow q$

$q \rightarrow r$

$\therefore p \rightarrow r$

b) $\neg p \wedge q \rightarrow \neg q$

c) $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$

EXERCISE 1O

1. Show that the following argument is valid or fallacy.

a) If daisy is a flower, then daisy is white

daisy is flower

∴daisy is white

b) If George does not have eight legs, then he is not an insect.

George is not an insect.

Therefore, George does have eight legs.

c) Linda is an excellent swimmer.

If Linda is an excellent swimmer, then she can work as a lifeguard.

∴Therefore, Linda can work as a

lifeguard.

d) If two sides of a triangle are equal, then the opposite angles are equal.

Two sides of a triangle are not equal.

∴The opposite angles are not equal.

EXERCISE 1P

1. What rules of inference is used in each of these arguments?

a) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.

b) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercise, I will understand the material. Therefore, If I work all night on this homework, then I will understand the material.

c) If it is snows today, the university will close. The university is not closed today. Therefore, it did not snow today.

EXERCISE 1Q

Test the validity of the following argument by using truth table. Then, conclude what is the rules of inference.

S_1: If a man is a bachelor, he is unhappy.

S_2: If a man is unhappy, he dies young.

S: Bachelors die young.

ADDITIONAL EXERCISES

1. Which of these sentences are propositions? State the truth values of those that are propositions.

a) Sunday is the day after Saturday.

b) I love teddy bear!

c) Is 2 is a positive number?

d) $2n + 3 > 6$, let $n = 2$

2. Let p, q, and r be the following statements.

p: You study hard;

q: You will get a good job;

r: You are happy.

Express each of these propositions into Logical Connectives.

a. If you study hard, then you will get a good job.

b. You are happy if and only if you study hard and you will get a good job.

c. If you do not study hard, you will not get a good job or not be happy.

3. Let P(x) be the statements “x can speak Russian” and let Q(x) be the statement “x knows the computer language C++”. Express each of these sentences in terms of P(x), Q(x), quantifiers and logical connectives. The domain for quantifiers consists of all students at your school.

a) All students at your school can speak Russian and knows C++.

b) There is a student at your school who can speak Russian but who doesn’t know C++.

4. What rule of inferences is used in each of these arguments?

a) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.

b) If it snows today, the children are happy. The children are not happy today. Therefore, it did not snow today.

c) It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.

5. Assuming p is “Amir will go to Japan” and q is “Amir intends to buy souvenirs.”. Express each of the following statements in English sentence form.

a) $q \vee \sim p$

b) $\sim p \wedge \sim q$

c) Show that $(p \vee q) \vee \sim p$ is a tautology.

6. Classify whether the following sentences are proposition or non-propositions.

a) Clean up your room.

b) $\mathrm{x} + 2 = 2\mathrm{x}, \text{when } \mathrm{x} = 2$

c) The product of 3 and 4 is 11.

7. Prepare a truth table for the proposition $\sim((p \wedge q) \wedge r)$ and show whether the proposition is tautology, contradiction or contingency.

8. Let $M(x, y)$ is a predicate "x has sent an email message to y" where the universe of discourse consists of all students in your class. Use quantifiers to express each of the following statements.

a) Alia has never sent an email message to Nurin.

b) Every student in your class has sent an email message to Sarah.

c) There is a student in your class who sent an email message to everyone in your class.

d) Every student in the class has sent an email message to some students in the class.

9. Assume $P(x)$ is the statement of “x is perfect” and $F(x)$ is “x is your friend”, whereby the domain of x consists of all people. Translate each of these statements into logical expressions using predicates, quantifiers and logical connectives

a) All people are not perfect.

b) At least one of your friends is perfect.

c) Not everybody is your friend or someone is not perfect.

CHAPTER 2

Boolean Algebra

BOOLEAN ALGEBRA

- BOOLEAN SUM
- BOOLEAN PRODUCT
- COMPLEMENTATION

COMPLEMENTATION

X	**X'**
0	1
1	0

BOOLEAN SUM

X	Y	F=X+Y
0	0	0
0	1	1
1	0	1
1	1	1

BOOLEAN PRODUCT

X	Y	F=X•Y
0	0	0
0	1	0
1	0	0
1	1	1

Example 1:

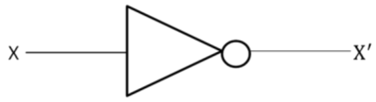
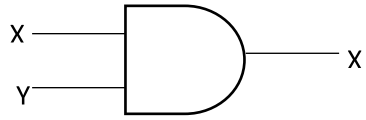
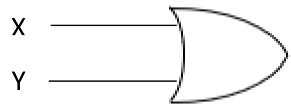
Find the value of
$1.0 + (\overline{0 + 1})$

Solution:
$1.0 + (\overline{0 + 1})$
$= 0 + (\overline{1})$
$= 0 + 0$
$= 0$

IDENTITIES OF BOOLEAN ALGEBRA

IDENTITY	NAME
$\overline{\overline{x}} = x$	**Law of the double complement**
$x + x = x$ $x.x = x$	**Idempotent laws**
$x + 0 = x$ $x.1 = x$	**Identity laws**
$x + 1 = 1$ $x.0 = 0$	**Domination laws**
$x + y = y + x$ $xy = yx$	**Commutative laws**
$x + (y + z) = (x + y) + z$ $x(yz) = (xy)z$	**Associative laws**
$x + yz = (x + y)(x + z)$ $x(y + z) = xy + xz$	**Distributive laws**
$\overline{(xy)} = \overline{x} + \overline{y}$ $\overline{(x + y)} = \overline{x}\,\overline{y}$	**De Morgan's laws**
$x + xy = x$ $x(x + y) = x$	**Absorption laws**
$x + \overline{x} = 1$	**Unit property**
$x\overline{x} = 0$	**Zero property**

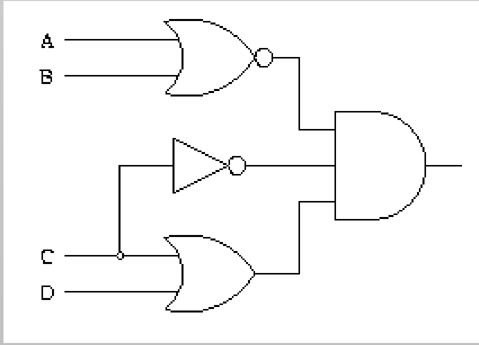
LOGIC GATES

1- The Inverter	1- The AND Gate	1- The OR Gate
 Performs the Boolean **NOT** operation.	 Performs the Boolean **AND** operation	 Performs the Boolean **AND** operation

Example 2 :

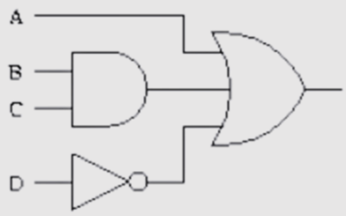
DRAW A LOGIC CIRCUIT CORRESPONDING TO THE BOOLEAN EXPRESSION

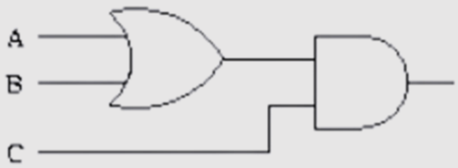
$(\overline{A + B})(C + D)\bar{C}$



Example 3:

EXPRESS THE OUTPUT FROM THE GIVEN CIRCUITS





$Y = A + BC + \overline{D}$

$Y = (A + B)C$

Example 4:

MINIMIZE BOOLEAN EXPRESSION USING KARNAUGH MAP

Minimize the following Boolean expression by using K-map.

$$F(x, y, z) = xyz + xy\bar{z} + x\bar{y}z$$

	$\boldsymbol{yz}$	$\boldsymbol{y\bar{z}}$	$\boldsymbol{\bar{y}\bar{z}}$	$\boldsymbol{\bar{y}z}$
$\boldsymbol{x}$	1	1	0	1
$\boldsymbol{\bar{x}}$	0	0	0	0

$$= \boldsymbol{xy + xz}$$

EXERCISE 2A

1. Find the values of these expressions.

a) $\mathbf{1 \cdot \overline{0}}$

b) $\mathbf{1 + \overline{1}}$

c) $\mathbf{\overline{0} \cdot 0}$

d) $\mathbf{\overline{(1 + 0)}}$

EXERCISE 2B

Find the truth table T for the equivalent Boolean expression

$$F(A, B, C) = ABC' + BC' + A'B$$

EXERCISE 2C

Simplify the following Boolean expression:

a) $C + \overline{BC}$

b) $\overline{AB}(\overline{A} + B)(\overline{B} + B)$

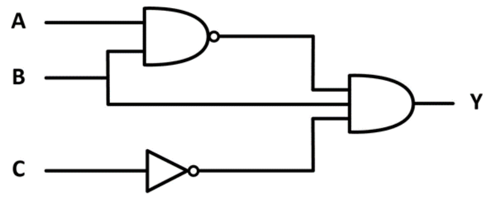
c) $(x + y)(xz + xz') + xy + y$

d) $\overline{x}(x + y) + (y + xx)(x + \overline{y})$

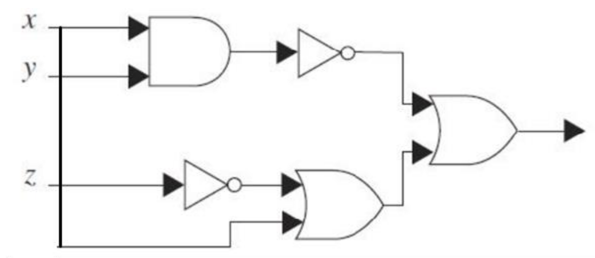
EXERCISE 2D

1. Find the output of the given circuits:

a)



b)



2. Draw a logic circuit corresponding to the Boolean expression:

a) $Y = (A+B)C$

b) $Y = A + BC + \overline{D}$

c) $Y=\overline{A+BC}+B$

d) $Y=\overline{A'B}+\overline{A+C}$

EXERCISE 2E

1. Here is a truth table for a specific three input logic circuit.

   Draw a K Map according to the values found in the truth table.

A	B	C	OUTPUT
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

2. Use Karnaugh maps to find the minimal form for each expression.

a) $xy + xy'$

b) $xy + x'y + x'y'$

c) $xy' + x'y'$

d) $xyz' + xy'z + xy'z' + x'yz + x'yz' + x'y'z$

e) $xyz + xyz' + x'yz + x'y'z$

f) $xyz + xyz' + xy'z + xy'z' + x'y'z$

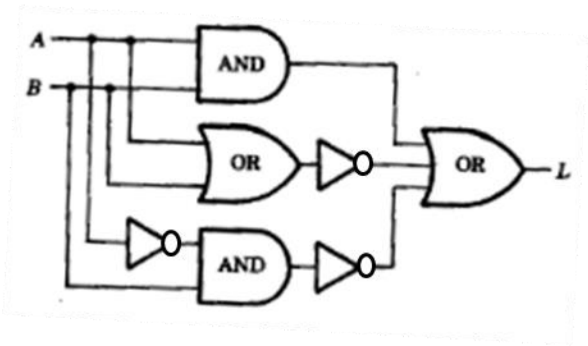
3. Design a minimal AND-OR circuit which yields the following truth table:

$$T = [A = 00001111,\ B = 00110011,$$
$$C = 01010101,\ L = 10101001]$$

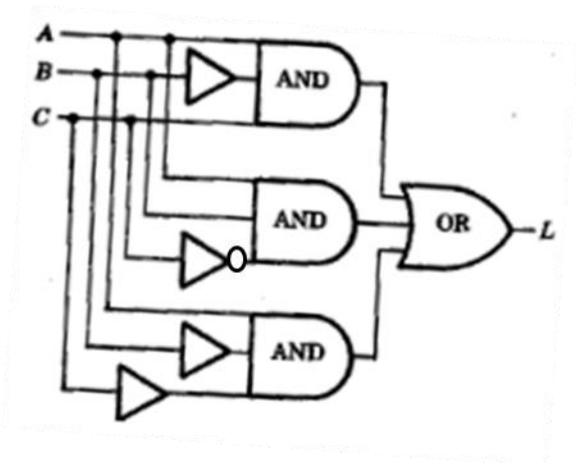
EXERCISE 2E

1. Redesign the following circuit so that it becomes a minimal AND-OR circuit

a)



b)



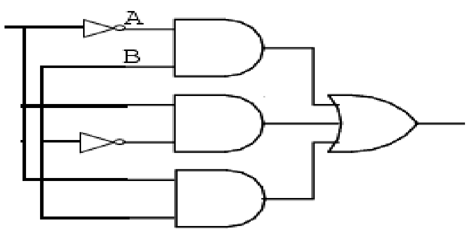
EXERCISE 2F

1. Simplify the following expression using a Karnaugh map:

i. $F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$

ii. $F(x,y,z) = \bar{x}yz + \bar{x}\bar{y}z + xy\bar{z} + \bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + xyz$

2. Consider the following circuit. Minimize the circuit using Karnaugh map.

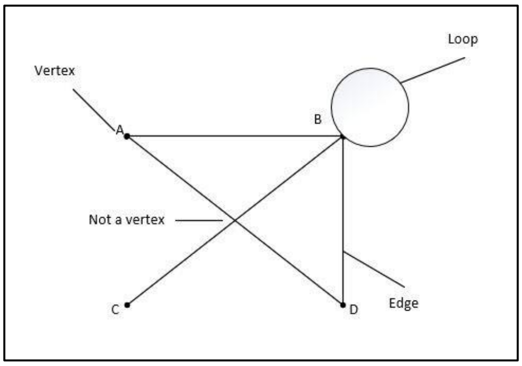


3. Simplify $F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}BC + \bar{A}B\bar{C} + ABC + AB\bar{C}$ using a Karnaugh map and draw the simplified circuit.

CHAPTER 3

Graphs And Trees

GRAPH TERMINOLOGY



SUMMARY OF GRAPH TERMINOLOGY IN GRAPH THEORY

The **size** of a graph is the number of its edges.

The **degree of a vertex** written $deg(v)$ is equal to the ***number of edges*** which are incident on v

The **sum of the degrees** of the vertices of a graph is equal to ***twice the number of edges***

The vertex v is said to be **even or odd (parity)** according as $deg(v)$ is even or odd

A vertex v is **isolated** if it is does not belong to any edge

A vertex with degree 1 is called a **leaf vertex**

The incident edge of vertex with degree 1 is referred as a **pendant edge**

A **path** is the sequence of connected vertices.

A **simple path** is a path where the vertices are only passed through once

A **trail** is a path where each edge is traveled once, meaning that there are no repeated edges (all edges are distinct)

The **length** of the path is the number n of edges that it contains

The **distance** between two vertices is described by the length of the shortest path that joins them

A **cycle / simple cycle** is a closed path with at least 3 edges, and no repeated vertices and

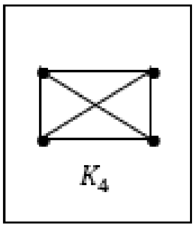
An **acyclic** is a graph that has no cycles in it

A **closed path or circuit** is a path that starts and ends at the same vertex

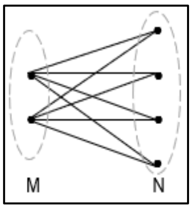
A graph is called **planar** if it can be drawn in the plane without any edges crossing each other

TYPE OF GRAPH

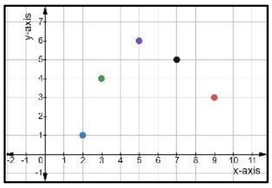
COMPLETE GRAPH



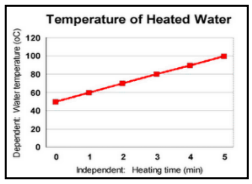
BIPARTITE GRAPH



DISCRETE GRAPH



LINEAR GRAPH

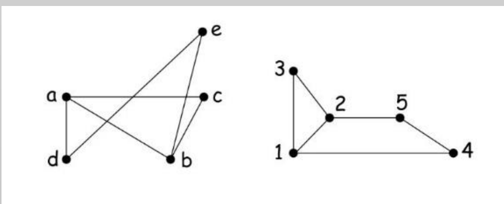


ISOMORPHIC GRAPH

Two or more graphs are isomorphic if they have the same:

- Number of vertices
- Number of edges
- Degree for each distinct vertices
- The graphs have bijective function

Example of isomorphic graphs:



GRAPH G

GRAPH H

Determine whether the graphs above are isomorphic or not?

Step		Graph G		Graph H	
1.	No. of vertices	5		5	
2.	No. of edges	6		6	
3.	Degree of vertex	Deg(a)	3	Deg(1)	3
		Deg(b)	3	Deg(2)	3
		Deg(c)	2	Deg(3)	2
		Deg(d)	2	Deg(4)	2
		Deg(e)	2	Deg(5)	2
4.	Illustrate the graph G & H to ensure it have bijective function! f(a)=1 f(b)=2 f(c)=3 f(d)=4 f(e)=5				

EULER PATH, EULER CIRCUIT, HAMILTON PATH, HAMILTON CIRCUIT

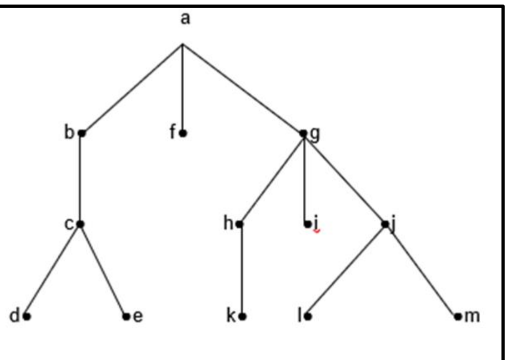
Euler path -A connected multigraph has an **Euler path** if and only if it has **exactly two vertices of odd degree**.

Euler Circuit- A connected multigraph has an **Euler circuit** if and only if **every vertex have even degree.**

Hamilton path-A **Hamilton path** is a simple path in a graph G that **passes through every vertex exactly once.**

Hamilton Circuit-A **Hamilton circuit** is a simple circuit in a graph G that **passes through every vertex exactly once.**

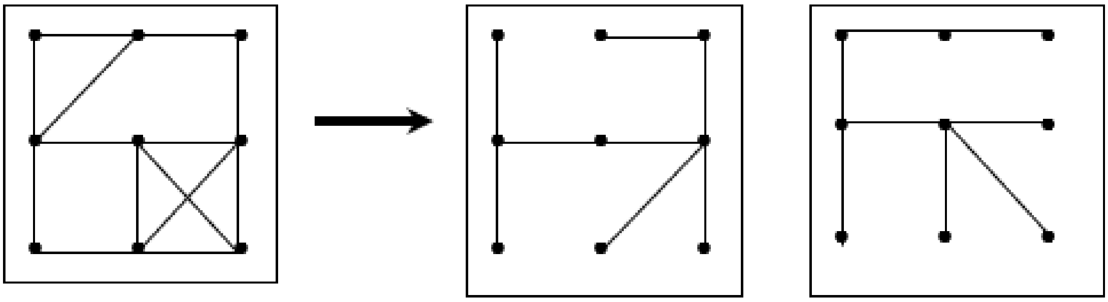
EXAMPLE OF TREES



A Rooted Tree

- The **root** is **a**.
- The **parent of h,i** and **j** is **g**.
- The **children of b** is **c.** The **children of j** are **l** and **m.**
- **h, i** and **j** are a **sibling**.
- The **ancestor of e** are **c, b** and **a**.
- The **descendants** of **b** are **c, d** and **e.**
- The **internal vertices** are **a, b, c, g, h** and **j.** (vertices that have children)
- The **leaves** are **d, e, f, i, k, l** and **m.** (vertices that have no children)

SPANNING TREES



MINIMAL SPANNING TREES

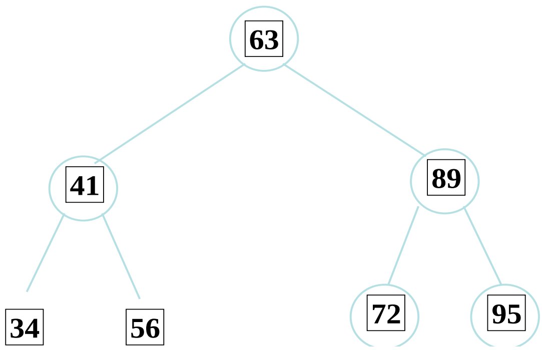
PRIM'S ALGORITHM

KRUSKAL ALGORITHM

EXAMPLES OF BINARY SEARCH TREE

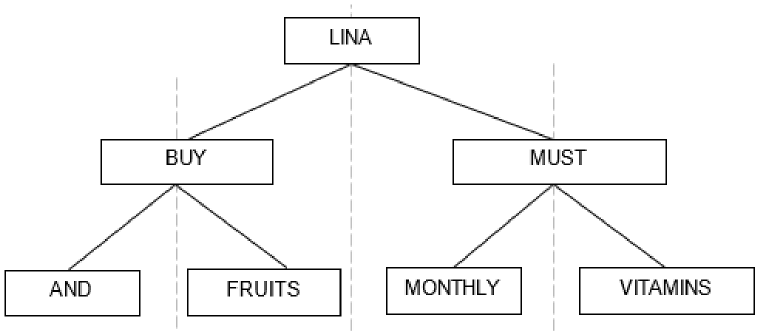
Example 1:

Construct a BST from set of data 63 89 72 41 56 95 34.



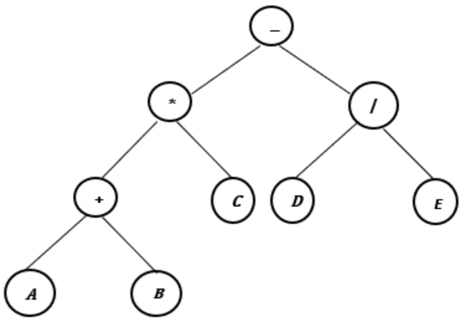
Example 2:

Construct a BST from the words "LINA MUST BUY FRUITS AND VITAMINS MONTHLY".



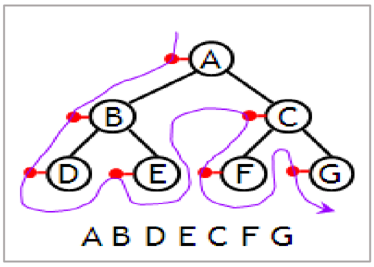
Example 3:

Construct a BST for the given arithmetic expression: $(A + B) * C - D/E$



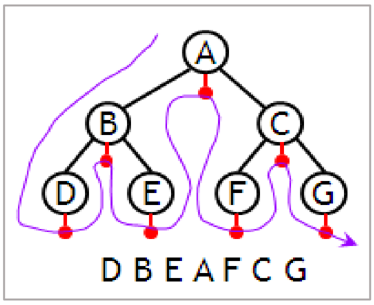
ORGANIZE TREE TRAVERSALS

PRE- ORDER TRAVERSAL:



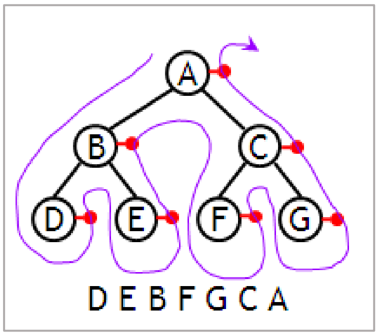
Pre-order: **A, B, D, E, C, F, G**

IN- ORDER TRAVERSAL:



In-order: **D, B, E, A, F, C, G**

POST- ORDER TRAVERSAL

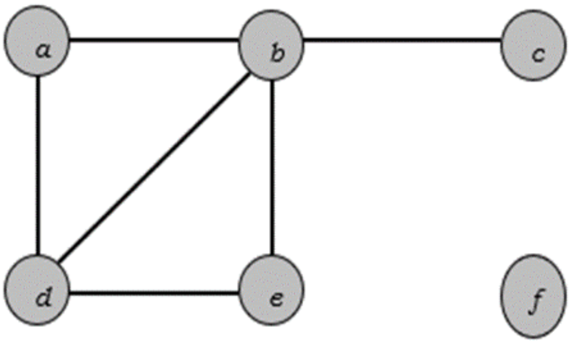


Post-order: D, E, B, F, G, C, A

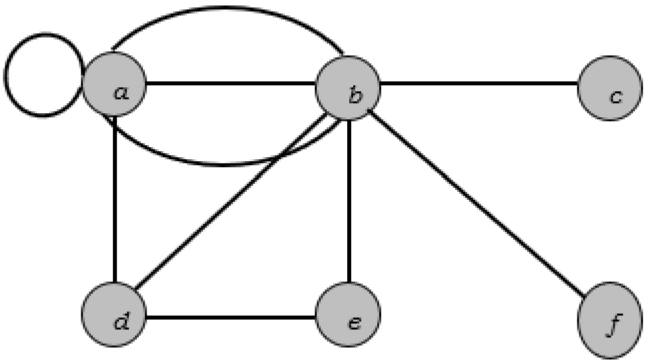
EXERCISE 3A

1. Find the **number of vertices**, the **number of edges**; identify all **parallel edges**, **loops** and **isolated vertices** for the following graph.

   a)

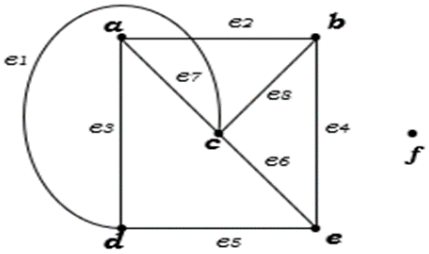


   b)



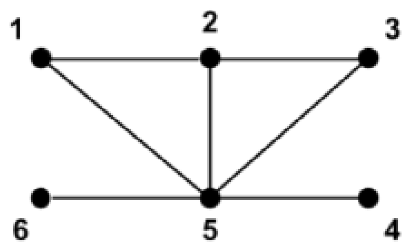
2. Find the **number of vertices**, the **number of edges**; identify all **parallel edges**, **loops** and **isolated vertices** for the following graph.

a)

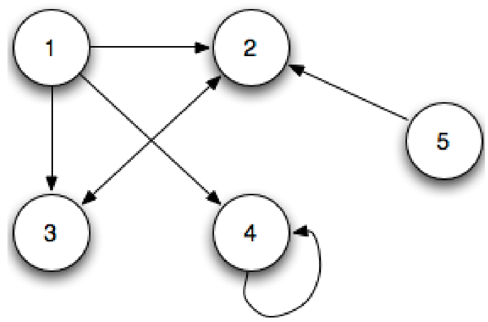


3. Find the degree of each vertex for the following graph.

a)



b)



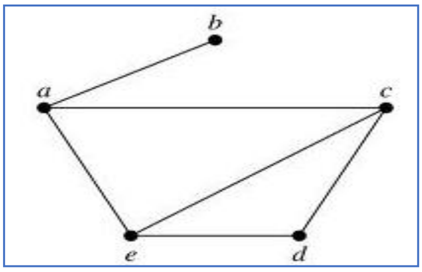
4. Draw a graph having the given properties or explain why no such graph exists.
   a) Six vertices each of degree 3

   b) Five vertices each of degree 2

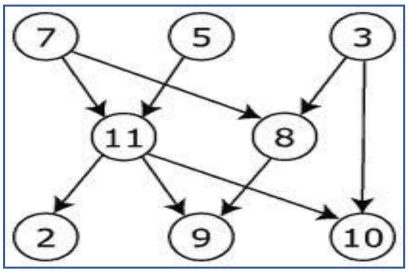
EXERCISE 3B

1. Draw a graph with the adjacency matrix $\begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$ with respect to the ordering of vertices a,b,c and d.

2. Use an adjacency matrix to represent the graph shown below.



3. From the graph shown below,



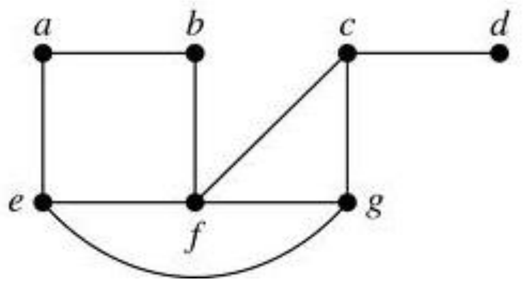
a) List down in a table form the in degree and the out degree of each vertices

b) Determine the parity (even or odd) for each vertex and

c) Determine the sum of degrees

EXERCISE 3C

From the graph below,



a) Determine the **parity** (even or odd) **for each vertex**

b) Identify the **leaf vertex**

c) Identify the **pendant edge**

d) Identify the **distance**, D from *a* to *g*

e) Identify the **size** of the graph

f) Find the 2-simple **path**, P_1, P_2 from *a* to *c*

g) Find a **trail**, T from *d* to *e*

h) Find 3 **simple cycle**, labeled as C_1, C_2 and C_3

EXERCISE 3D

1. Calculate the number of edges in complete graph $\mathbf{K_8}$.

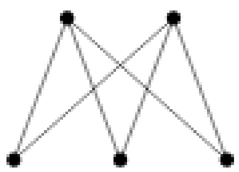
2. How many vertices and edges are in K_{100}?

3. Draw Complete bipartite graph $\mathbf{K_{3,3}}$.

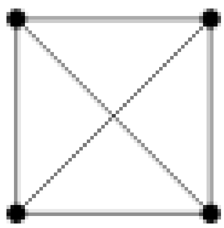
EXERCISE 3E

Is this a planar graph or not?

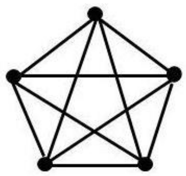
a)



b)



c)

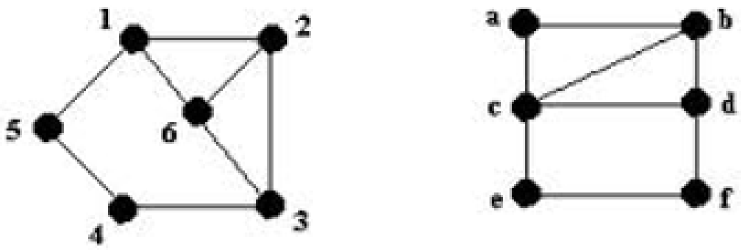


K_5

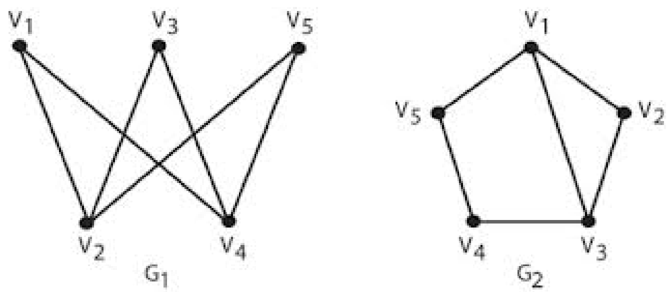
EXERCISE 3F

1. Determine whether the graphs below are isomorphic or not.

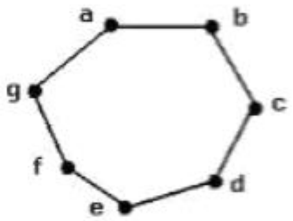
a)

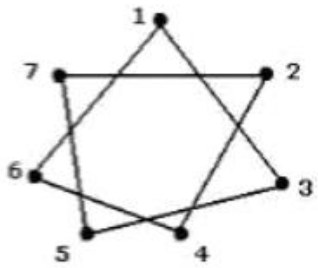


b)

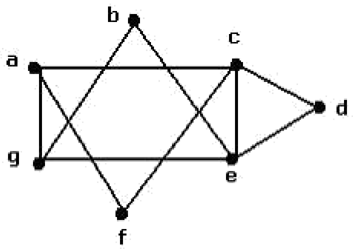


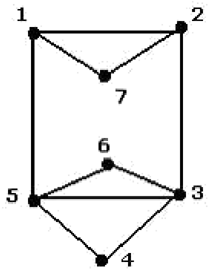
c)





d)

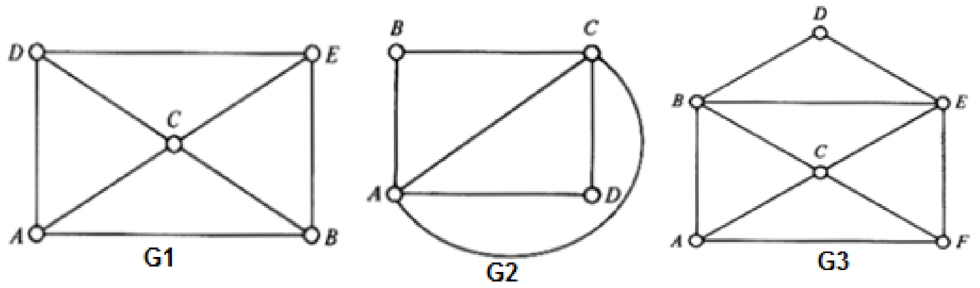




EXERCISE 3G

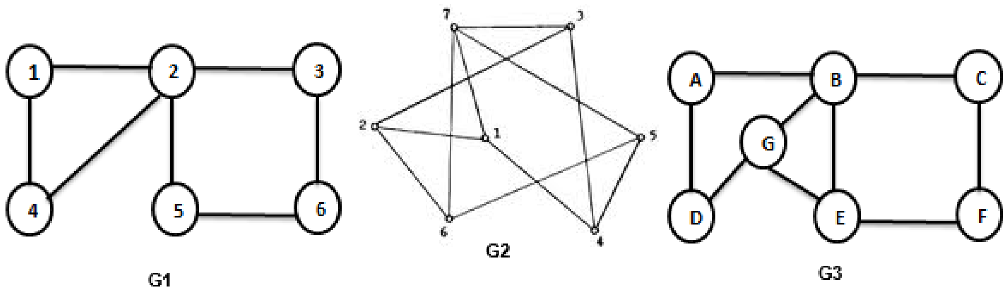
1. Give the characteristic of Euler path and Euler circuit.

2. Which of the following graphs G1, G2 and G3 has Euler path or Euler circuit? If any, list down the Euler path or Euler circuit.



EXERCISE 3H

1. Which of the following graphs G1, G2 and G3 has **Hamilton path or Hamilton circuit**? If any, list down the Hamilton path or Hamilton circuit.



EXERCISE 3I

1. How many edges does a full 3-ary tree with 500 internal vertices have?

2. How many leaves does a full binary tree with 1001 vertices have?

EXERCISE 3J

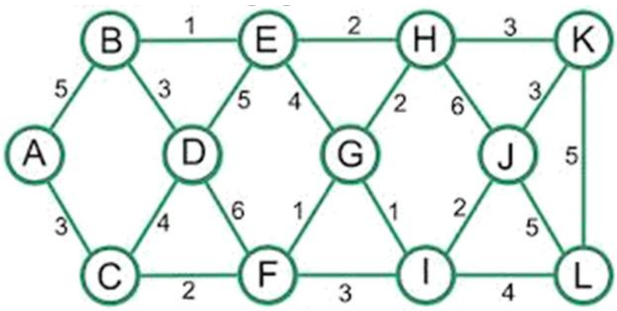
1. Draw a rooted tree having the following properties:

   a) Full binary tree, four internal vertices and five terminal vertices (leaves)

   b) Full binary tree, six internal vertices and seven terminal vertices

EXERCISE 3K

1. Based on the following figure,



a) Draw 2 possible spanning trees

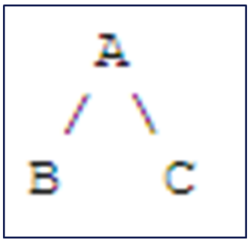
b) Find a minimum spanning tree by using Kruskal's and Prim's algorithms (compare the findings)

c) Draw the minimum spanning tree.

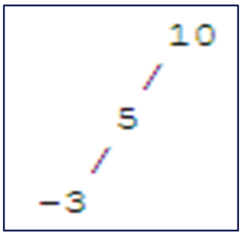
EXERCISE 3L

1. Which of the following binary trees are BSTs? If a tree is **not** a BST, why?

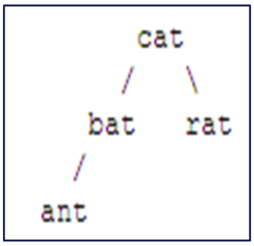
(a)



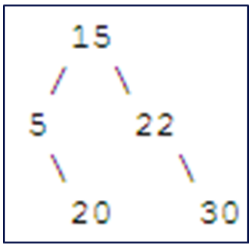
(b)



(C)



(d)



EXERCISE 3M

1. Create a BST from the following set of data, with the first number is the root

   a) 14 4 17 19 15 7 9 3 16 10

b) 9 23 17 4 30 7 14 16 15 19

c) 19 27 16 15 14 30 39 17 9 3

2. Create a BST from the following sequence of words (sentence), with the first word is the root.

   a) MOST STUDENTS IN THIS CLASS HAS STUDIED BASIC LOGIC

b) EVERYTHING HAPPENS FOR A REASON, EVEN WHEN WE ARE NOT WISE TO SEE IT

3. Represent the following arithmetic expression as binary tree:

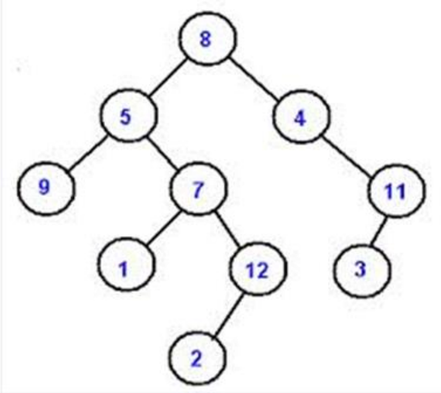
a) $(\mathrm{A}+\mathrm{B}) * (\mathrm{C}-\mathrm{D})$

b) $\big(((\mathrm{A}+\mathrm{B}) * \mathrm{C}+\mathrm{D}\big) * \mathrm{E}) - ((\mathrm{A}+\mathrm{B}) * (\mathrm{C}-\mathrm{D}))$

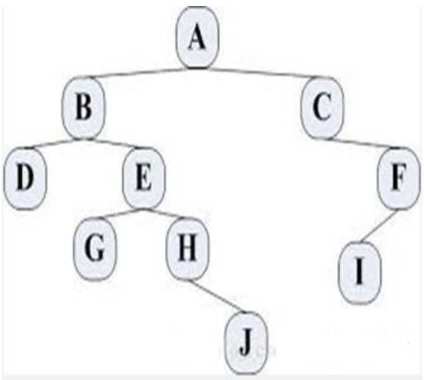
EXERCISE 3M

1. Find the tree traversals (pre-order, in-order & post-order) of the following tree:

a)



b)



ADDITIONAL QUESTIONS

Answer all the questions:

1. Construct the binary trees for the algebraic expression below:

$$(x+(y-(x+y)))*((3\div(2*7))*4)$$

2.

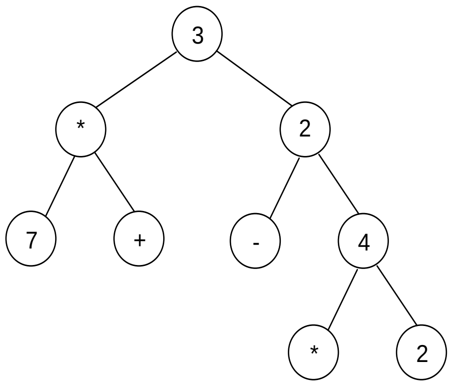


Figure 1

Based on the rooted tree in Figure 1, find:

a) a **preorder** traversal

b) an **inorder** traversal

c) a **postorder** traversal

3. Sketch the following graphs.

a) A complete graph of K_5.

b) A complete bipartite graph of $K_{2,3}$.

c) A linear graph of L_5.

d) A discrete graph of D_5.

4. Identify whether a complete graph of K_5 is a planar or not. Explain your answer.

5. Refer to Figure 2, determine whether Graph A is isomorphic to Graph B or not. Explain your answer by giving three reasons.

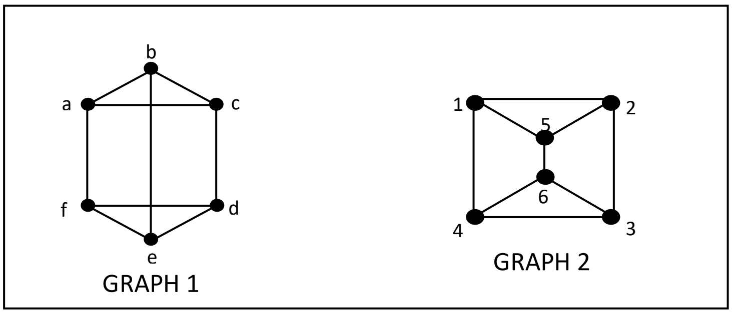


FIGURE 2

6. Determine whether the given graph in Figure 3 and Figure 4 has an Euler circuit or not. Construct such a circuit if it exists. If no Euler circuit exists, determine whether the graph has an Euler path or not. Construct such a path if exists.

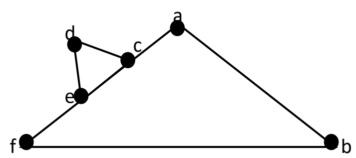
a) 

Figure 3

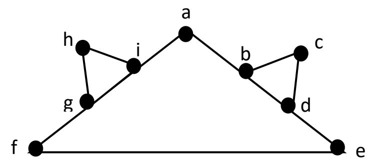
b) 

Figure 4

7. Determine whether the given graph in Figure 5 and Figure 6 has a Hamilton circuit or not. Construct such a circuit if exists. If no Hamilton circuit exists, determine whether the graph has a Hamilton path or not. Construct such a path if exists.

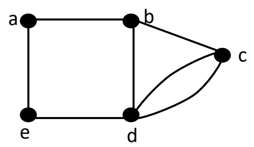
a) 

Figure 5

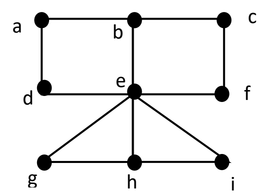
b) 

Figure 6

8. Answer these questions about the rooted tree as illustrated in Figure 7.

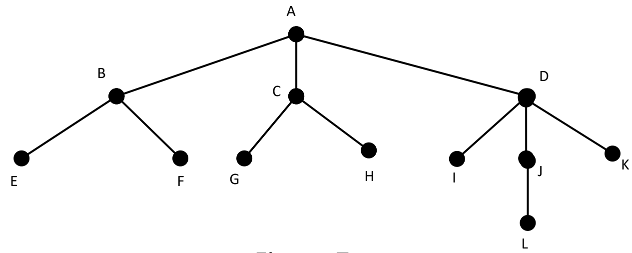


Figure 7

a) Which vertex is the root?

b) Which vertices are leaves?

c) Which vertices are children of C?

d) Which vertices are siblings of J?

9. Find two (2) possible spanning trees for the following weighted graph in Figure 8.

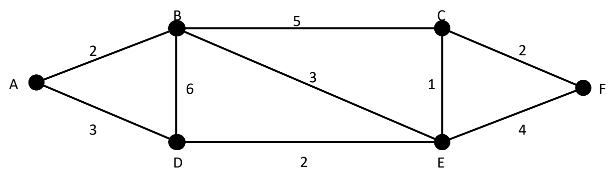


Figure 8

10. The Roads represented by the graph in Figure 9 are all unpaved. The lengths of the roads between two towns are represented by edge weights. Using Prim's algorithm, which road should be paved so that there is a path with a minimum road length? (Begin at Alor Setar and end at Jitra)

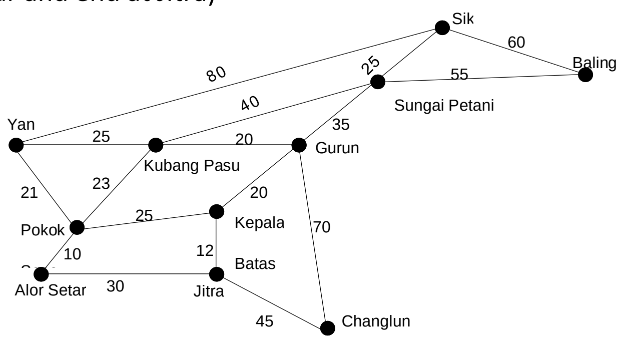


Figure 9

11. Built a binary search tree for the word Firdaus, Adn, Naim, Mawa, Darus Salam, Darul Muqamah, Amin, Khuldi using alphabetical order.

12. For the ordered rooted tree in Figure 10, determine the order in which a pre-order, in-order and post-order traversals visit the vertices.

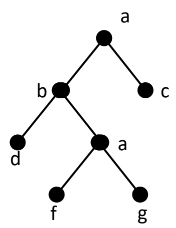


Figure 10

13. Consider the following graphs in Figure 11

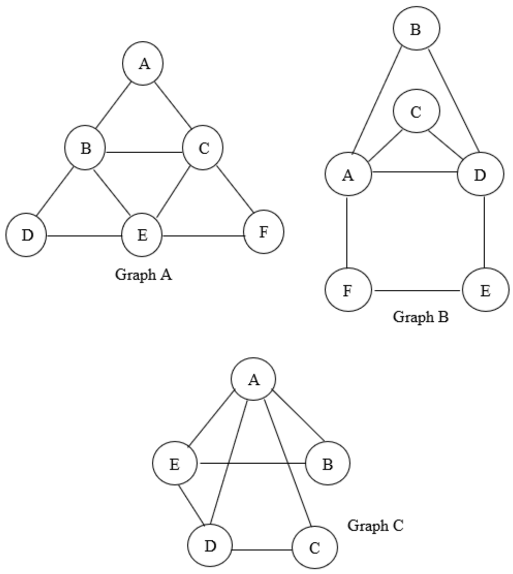


Figure 11

a) Determine which graphs has the Euler circuit. Explain your answer. Construct the circuit if it exists.

b) Determine which graphs has the Hamilton path. Explain your answer Construct the path if it exists.

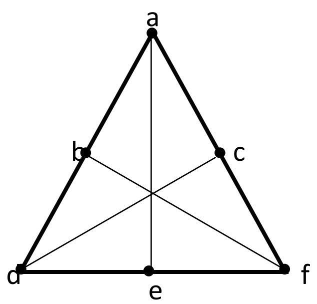
14. Give the correct answer for each of the following statement.
    a) A graph with a number is assigned to each edge.

    b) A path that begins and ends at the same vertex.

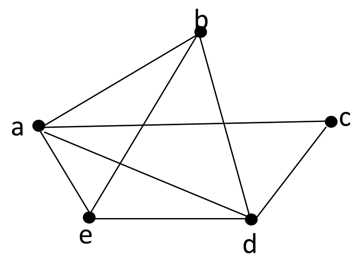
    c) An undirected graph with multiple edge and loop.

    d) A vertex with degree zero

15. By referring to Figure 12, determine whether Graph A and Graph B are planar or not. Explain your answer by redraw the graph.



Graph A



Graph B

Figure 12

16. State the suitable graph terminology for each of the following statements.
    a) A graph with numbers on the edges

    b) A graph without any loop and parallel edges

    c) A Graph with loop and parallel edges

    d) A vertex of degree zero

    e) A vertex with degree one

17.

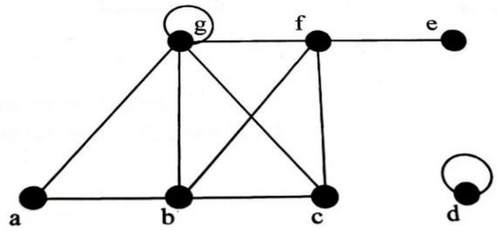


Figure 13

Based on Figure 13
    a) Identify leaf vertex

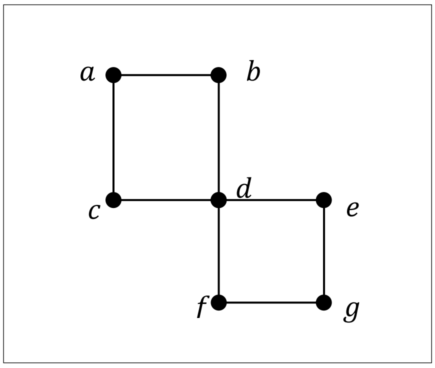
    b) Identify pendant edges

    c) Identify the loops.

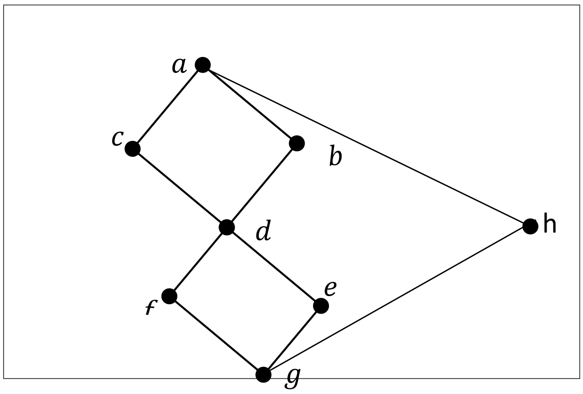
    d) How is the size of the graph is being determined and what is the size of the graph?

    e) State the degree of vertices a,b,c and f.

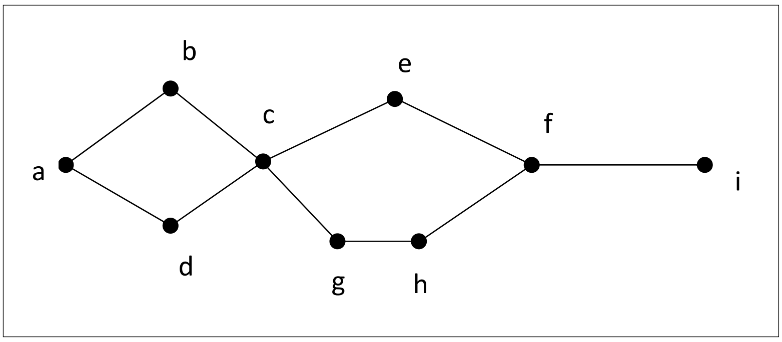
18.



Graph X



Graph Y



Graph Z

Consider the graphs above:

i. Determine which graph has the Euler circuit and then determine which graph has the Euler Path. Explain why the graph has Euler path or Euler circuits.

ii. Construct the circuit and path in (i) if it exists.

19. State the terminology for the following definitions.

a) A path that starts and ends at the same vertex.

b) The edge linking vertex to itself.

c) The path with no repeated vertices.

d) A vertex with degree 1.

e) A path contains at most two vertices of odd degree.

20. Consider the following graphs in Figure 14

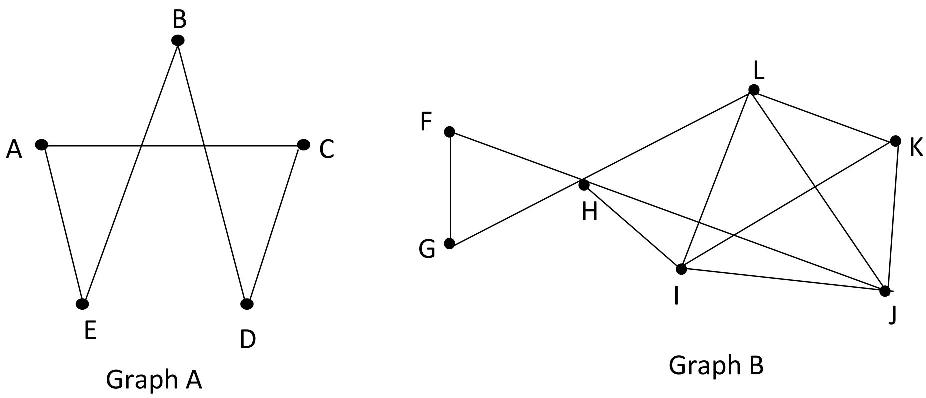
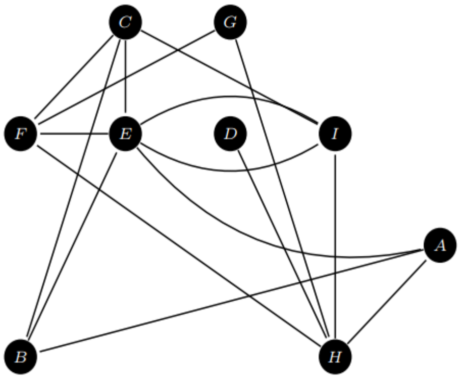


Figure 14

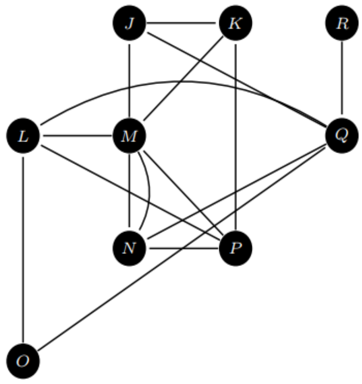
a) Determine which graphs have a Euler circuit. Explain your reason. Construct two different circuits if it exists.

b) Determine which graphs have a Hamilton path. Explain your reason. Construct two different paths if it exists.

21. Determine whether graphs in Figure 15 are isomorphic or not. Justify your answer.



Graph P



Graph Q

Figure 15

22. Determine whether the graph in Figure 16 below is a planar or not. If it is a planar, redraw the graph. If it is not a planar, explain your answer.

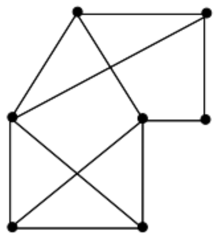


Figure 16

CHAPTER 4

Sets, Relations And Functions

EQUAL SETS

The set are equal if and only if their number of elements and the member of elements are exactly same.

Example of equal sets: **A = {5,6,7}, B = {7,5,6}, C = {5,5,6,6,7,7}**

SPECIAL SYMBOLS FOR SETS

N = the set of **positive integers** : 1, 2, 3, ….

Z = the set of **integers** : ….., -2, -1, 0, 1, 2, …..

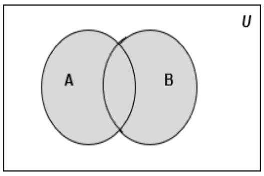
Q = the set of **rational numbers**

R = the set of **real numbers**

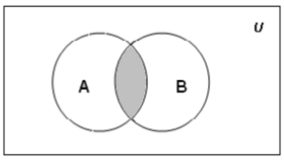
C = the set of **complex numbers**

OPERATION ON SETS

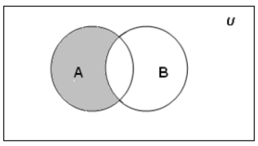
$A \cup B$



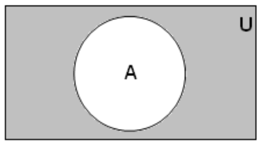
$A \cap B$



$A - B$



A'

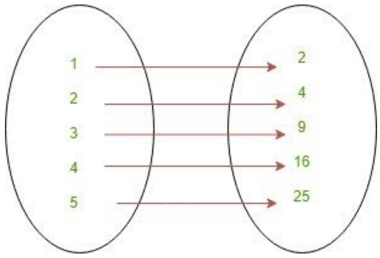


RELATION REPRESENTATION

GRAPHICALLY/ ARROW DIAGRAM

Example:

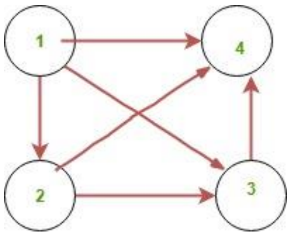
R= {(1, 1), (2, 4), (3, 9), (4, 16), (5, 25)}



DIGRAPH (DIRECTED GRAPH)

Example:

R= {(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)}



MATRICES

Example:

Suppose that A = {1,2,3} and B = {1,2}. R= {(2, 1), (3,1), (3,2)}

In matrix form;

$$\boldsymbol{M_R} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} \end{bmatrix}$$

PROPERTIES OF RELATIONS

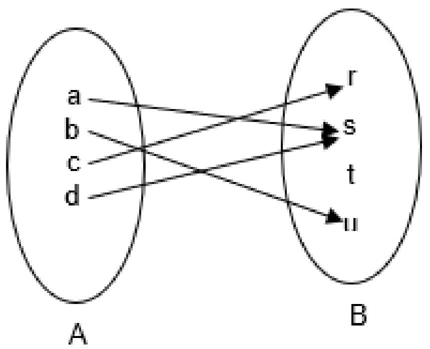
- **REFLEXIVE**
- **SYMMETRIC**
- **TRANSITIVE**

*A relation on a set A is called an **equivalence relation** if it is *reflexive*, *symmetric* and *transitive*.

IMPORTANT TERMS USED IN FUNCTIONS

- The element in set A is called the ***domain***
- The element in set B is called ***codomain***
- Unique element of B which is assign to A is called ***image / range.***

Example 1:



Domain = {a, b, c, d}

Codomain = {r, s, t, u}

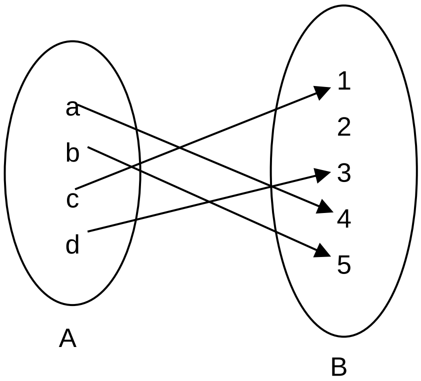
Range/Image = {r, s, u}

ONE-TO-ONE FUNCTIONS

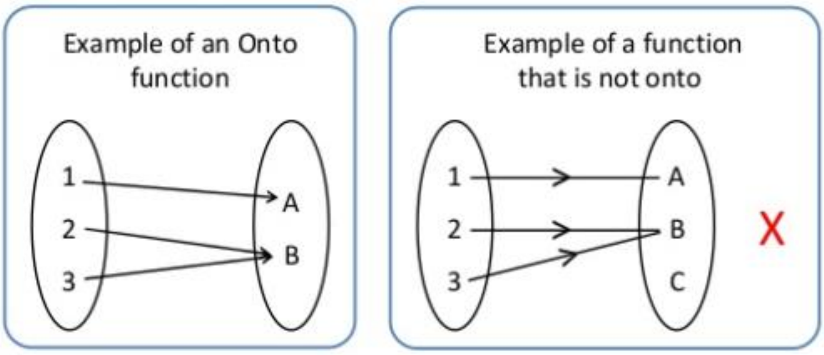
Example 2:

Determine whether the function f from {a, b, c, d} to {1, 2, 3, 4, 5} with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$ and $f(d) = 3$ is one-to-one.

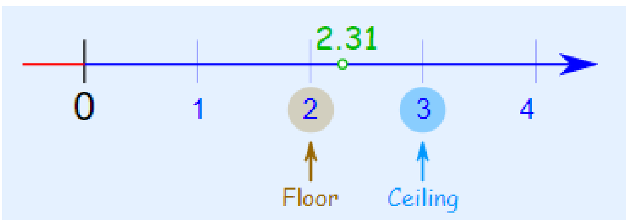
Solution: The function f is one-to-one.



ONTO FUNCTION



DESCRIBE FLOOR AND CEILING FUNCTIONS



Notes: $\lfloor 2.31 \rfloor = 2$
$\lceil 2.31 \rceil = 3$

Another example: Solve $\lfloor 0.5 + \lceil 1.3 \rceil - \lceil -1.3 \rceil \rfloor$
$= \lfloor 0.5 + 2 - (-1) \rfloor$
$= 3$

EXERCISE 4A

1. List the elements of the following sets; here $N = \{1,\ 2,\ 3\ \ldots\}$.
   a) $A = \{x : x \in N,\ 3 < x < 10\}$

   b) $B = \{x : x \in N,\ x\ is\ even,\ x < 15\}$

   c) $C = \{x : x \in N,\ 4 + x = 7\}$

2. Given the universal set) $U = \{x : 15 \leq x \leq 26,\ x\ is\ an\ integer\}$,

   $M = \{x : x\ is\ a\ prime\ number\}$; $N = \{x : x\ is\ a\ multiple\ of\ 3\}$; $O = \{x : x\ is\ an\ even\ number\}$

   List the elements of set M, N and O.

3. List the element of the following sets:
   a) {x: $x \in$ N, 5 < x < 12}, where N = {1, 2, 3…}

   b) {x: $x \in$ N, x is even, x < 15}, where N = {1, 2, 3…}

   c) {*x*: $x \in$ N*,* 10 < *x* < 35, *x* with sum of digits less than 6} where N = {1, 2, 3…}

1. Determine whether each pair of sets is equal
   a) {1, 2, 2, 4}, {1, 2, 4}

   b) {1, 1, 3}, {3, 3, 1}

   c) $\{x \mid x^2 + x = 2\}$, {1, -1}

   d) $\{x \mid x \in \mathrm{R} \text{ and } 0 < x \leq 2\}$, {1, 2}

EXERCISE 4B

1. Consider the following sets:

$\emptyset$, $A = \{1\}$, $B = \{1, 3\}$, $C = \{1, 5, 9\}$, $D = \{1, 2, 3, 4, 5\}$,

$E = \{1, 3, 5, 7, 9\}$, $U = \{1, 2, \ldots\ldots, 8, 9\}$

Insert the correct symbol $\subseteq$ or $\nsubseteq$ between each pair of set.

a) $\emptyset$, A b) A , B c) B , C d) B , E

e) C , D f) C , E g) D , E h) D , U

2. Use a Venn Diagram to illustrate the relationship
   a) $A \subseteq B$ and $B \subseteq C$.

b) $A \subseteq D$ and D$\subseteq U$

EXERCISE 4C

1. Given $A = \{1,\ 2,\ 3,\ 4,\ 5\}$, $B = \{4,\ 5,\ 6,\ 7\}$, $C = \{5,\ 6,\ 7,\ 8,\ 9\}$,

   $D = \{1,\ 3,\ 5,\ 7,\ 9\}$, $E = \{2,\ 4,\ 6,\ 8\}$, $F = \{1,\ 5,\ 9\}$.

Find:

a) $A \cup B$

b) $B \cap D$

c) $D \cap E$

d) $F \backslash A$

e) $C \cup F$

f) $E - B$

g) $D \backslash A$

2. Let $A = \{a,\ b,\ c,\ d,\ e\}$ and $B = \{a,\ b,\ c,\ d, e,\ f, g,\ h\}$. Find:

a) $A \cup B$

b) $A \cap B$

c) $A - B$

d) $B - A$

3. The universal set,

   U = {x: 10 < x < 35, x is an integer},

   F = {x: x is a prime number},

   G = {x: x is a multiple of 3}

   H = {x: x < 20}.

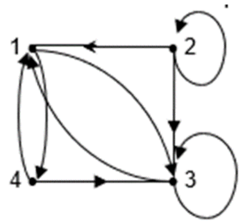
a) List all the elements of set F, G and H.

b) Draw the Venn diagram to represent all elements of **F ∪ G ∪ H** in the universal set.

c) Find **n(F ∩ H)**

EXERCISE 4D

1. What are the ordered pairs in the relation R represented by the directed graph shown below?



2. Draw a directed graph (Digraph) for the relation

   a) R = { (1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)} on the set {1, 2, 3, 4}

   b) R = {(a,b), (a,a), (b,b), (b,c), (c,c), (c,b), (c,a)} on the set {a, b, c}

3. **List** and display all the relation **graphically** the ordered pairs in the relation R from A = {0, 1, 2, 3, 4} to B = {0, 1, 2, 3} where (a, b) $\in$ R if and only if

   a) a = b

b) $a + b = 4$

c) $a > b$

d) a divides b ($a \mid b$) *(*it means b / a)*

State the **domain and range** for all the questions above.

EXERCISE 4E

1. Consider the relation on the set { 1, 2, 3, 4, 5}:R = { (1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,2), (4,4), (5,1), (5,3), (5,5) }, determine whether the relation R is reflexive, symmetric or transitive. Explain your answer.

2. Let M = {0, 1, 2,3} and defined relation R = {(0,1), (0,3), (1,0), (1,1), (2,3), (3,0), (3,2), (3,3)}

   a) Represent the relation R using directed graph

   b) Determine whether the relation R is reflexive, symmetric or transitive. Explain your answer.

3. Given those three relations on set A = {1,2,3,4}:

   R = {(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,1),(4,4)}

   S = {(1,1),(1,3),(1,4),(3,4)}

   T = {(1,2),(2,2),(2,3)}

   a) Determine which relations are reflexive. Give your reason.

   b) Determine which relations are not symmetrical. Explain your answer.

   c) Give a reason why S is transitive.

4. Given A = {1,2,34}, B = {1,4,6,8,9} where element a is in A is related to element b in B, if and only if $b = a^2$

   a) List the element of the relation

   b) Draw the directed graph for the relation

   c) Determine whether the relation is reflexive or not.

   d) Is the relation symmetric? Explain your answer.

EXERCISE 4F

1. Given g = {(1,b), (2,c), (3,a)}, a function from X = {1, 2, 3} to Y = {a, b, c, d} and

   f = {(a,x), (b,x), (c,z), (d,w)}, a function from Y to Z = {w, x, y, z}, write $f \circ g$ as a set of ordered pairs and draw the arrow diagram of $f \circ g$.

2. Let f and g be functions from the positive integers to the positive integers defined by equations $f(n) = 2n + 1$, $g(n) = 3n - 1$. Find the compositions $f \circ f$, $g \circ g$, $f \circ g$, *and* $g \circ f$.

3. Given that the function $f(x) = 4x + 1$, find a formula for $f^{-1}(x)$.

4. Given that the functions $g(x) = x - 2$ and $f(g(x)) = x^2 - 4x + 8$. Find:

   a) g (8)

   b) The values of x if $f(g(x)) = 13$

   c) The function $f(x)$

   d) $g^{-1}(-1)$

5. Given that $f(x) = 2x + x^2$ and $g(x) = 1 - \frac{x}{4}$. Find:

   a) f (3)

   b) fg (– 4)

EXERCISE 4E

1. Find these values.

   a) $\lfloor -1 \rfloor$

   b) $\lceil 3 \rceil$

   c) $\lfloor \frac{1}{4} + \lceil \frac{5}{4} \rceil \rfloor$

   d) $\lfloor 8.9 + 0.7 \rfloor$

   e) $\lfloor 8.9 \rfloor + \lfloor 0.7 \rfloor$

   f) f) $\lfloor 0.5 + \lceil 1.3 \rceil - \lceil -1.3 \rceil \rfloor$

g) $\lceil \lfloor 1.6 \rfloor + 2.3 - \lceil 1.1 \rceil \rceil$

2. Calculate the value of $\lceil 3.7 \rceil - \lceil 1.2 + \lfloor 2.5 \rfloor \rceil + \lfloor 4.2 \rfloor$.

ADDITIONAL EXERCISE

1. Given $\xi = P \cup Q \cup R$ where

$$\xi = \{x : 3 \leq x \leq 10\}$$
$$P = \{x : x \text{ is a prime number}\}$$
$$P \cup Q = \{x : x \text{ is an odd number}\} \text{ and } P \subset Q$$

   a) list the elements of:

      i) set Q

      ii) set $Q' \cup P$

   b) find $n(P' \cap Q)$

2. Given the universal set U = {a, b, c, d, e, f, g, h}, set R = {a, b, c, d, e, f}, set S = {c, d, e} and set T = {f, g, h}.

   a) Construct a Venn diagram illustrating the sets.

   b) What is the relation between set R and set S?

   c) Give the element for S’ U T.

   d) Find n(R∩T’).

3. Given that the functions $g(x) = x - 2$ and $fg(x) = x^{2}-4x+32$. Find

a) The values of x if $g(x) = 20$

b) The function f

c) $g^{-1}(6)$

4. Given the function $f(x) = 6x + 4$ and $g(x) = 2x^2 + 3x$.

   a) Calculate $f(4)$.

   b) Determine $fg(x)$ and $gf(x)$.

   c) Determine $f^{-1}(5)$.

5. Find these values:

   a) $\lfloor \frac{1}{2} - \lceil \frac{7}{2} \rceil + \lfloor 4 \rfloor \rfloor$

   b) $\lfloor 0.6 + \lceil -3.2 \rceil - \lceil 3.2 \rceil \rfloor$

   c) $\lceil 6.8 \rceil - \lceil 1.7 + \lfloor 4.3 \rfloor \rceil + \lfloor 0.2 \rfloor$

6. Let $A = \{a, b, c, d\}$ and $B = \{1,3,5\}$.Given that the function $f = \{(a, 1), (b, 3), (c, 5), (d, 5)\}$.

   a) Sketch the arrow diagram of the function *f.*

   b) State whether the function *f* is one to one or onto function.

7. Given the function $f(x) = x - 5$ and $g(x) = 3x^2 - 2$. Determine:

   a) The value of $g(2)$

   b) The value of $fg(x) = 5$

8. Find the floor and ceiling values for the following numbers.

   a) $\left\lceil \frac{1}{2} \cdot \left\lfloor \frac{2}{5} \right\rfloor + \lfloor 3.7 \rfloor \right\rceil$

   b) $\lceil 1.5 + \lfloor 0.2 \rfloor — 4.3 \rfloor$

   c) $\left\lceil \left\lceil \frac{1}{4} + \frac{1}{3} \right\rceil - \lfloor -0.5 \rfloor + \left\lfloor 2 \cdot \frac{1}{3} \right\rfloor \right\rceil$

9. Consider the following:

A is the set of distinct letters in the word *UNIVERSITY*

B is the set of distinct letters in the word *DENSITY*

C is the set of distinct letters in the word *MATHEMATICS*

D is the set of distinct letters in the word *PROGRAMMING*

The universe U is the set of all upper-case letters of the English alphabet. Determine all the elements of:

a) $A \cap C$

b) $A \cap (C \cup D)$

c) $n(A \cup B \cup C \cup D)'$

10. Given the function $g(x) = 5x + 7$ and $gf(x) = 7x - 9$. Determine:

a) The value of x if $g(x) = -8$

b) The function of $f(x)$

c) $g^{-1}(3)$

11. Find the floor and ceiling values for the following numbers.

a) $\left\lceil \left\lceil \frac{3}{4} \cdot \left\lceil \frac{2}{7} \right\rceil + \lfloor 7.9 \rfloor \right\rceil \right\rceil$

b) $\left\lceil 4.8 + \lfloor -0.3 \rfloor - \lceil -2.6 \rceil \right\rceil$

c) $\left\lceil \left\lceil \frac{1}{8} + \frac{7}{12} \right\rceil - \lceil -0.3 \rceil + \left\lfloor 27 \cdot \frac{1}{9} \right\rfloor \Big| \right\rceil$

12. Given that the universal set $\xi = \{x: 12 \leq x \leq 25, x\ is\ an\ integer\}$,

    Set $P = \{13,15,16,18\}$,

    Set $Q = \{x: x\ is\ a\ prime\ number\}$ and

    Set $R = \{x: x\ is\ and\ odd\ number\}$.

    a) Find the element for the set $(\mathrm{P} \cup \mathrm{Q})' \cap \mathrm{R}$.

    b) Sketch the Venn diagram for the set $\xi, \mathrm{P}, \mathrm{Q}$ and R.

13. Given the function $g(x) = \sqrt{(x-1)}$ and $f(x) = x^2 + 1$.

    a) Calculate $f(2) - g(17)$.

    b) Determine $gf(x)$.

    c) Determine $f^{-1}(5)$.

14. Find the values for the following:

a) $\lfloor \lfloor 1.4 \rfloor + 3.5 - \lceil 1.2 \rceil \rfloor$

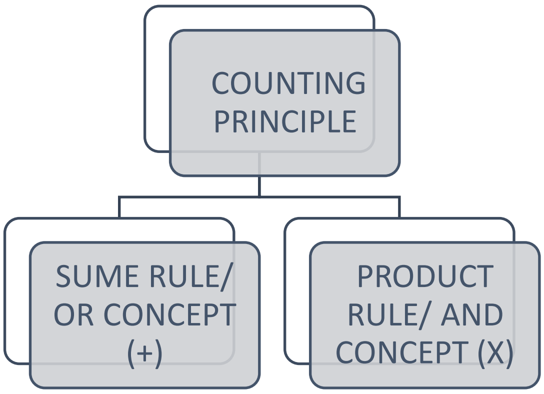
b) $\lfloor 0.6 \rfloor + \lceil -3.2 \rceil - \lceil 3.2 \rceil$

c) $\left\lceil 2 \left\lfloor \frac{1}{2} \right\rfloor + \lceil \sqrt{5} \rceil - \left\lfloor \frac{1}{2} + \frac{1}{2} \right\rfloor \right\rceil$

CHAPTER 5

Basic Counting Rules

BASIC COUNTING RULES



COMBINATION OF SUM AND PRODUCT RULE

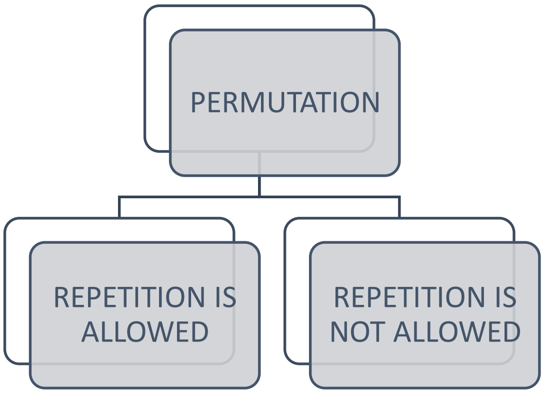
Example 1: Pastry shop menu:

6 kinds of muffins, 8 kinds of sandwiches, hot coffee, hot tea, ice tea, cola, orange juice

Buy either a muffin and a hot beverage, or a sandwich and a cold beverage. How many possible purchases?

Solution: $(6 \times 2) + (8 \times 3) = 36$

PERMUTATION



REPETITION IS ALLOWED

Example:

How many different passwords can be made if each password contains a sequence of three letters followed by three digits? (Note: Repetition of English letters and digits are allowed)

Solutions:

$$26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17576000$$

REPETITION IS NOT ALLOWED

Example 1:

There are 16 balls tagged with number 1 till 16. How many ways can we pick **3 balls without repeating the same balls.**

Solutions:

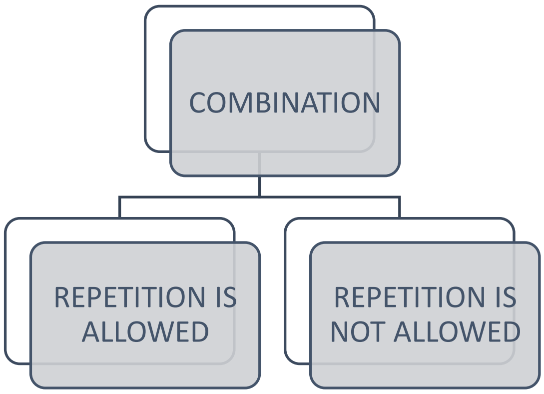
$$^{16}P_3 = P(16,3) = \frac{16!}{(16-3)!} = \frac{16!}{(13)!} = \textit{3,360 ways}$$

Example 2: Find the number of arrangements for the word BENZENE

Solutions: BENZENE has repetition letters which are E & N. E is repeated 3 times and N is repeated 2 times.

$$P(7;3,2) = \frac{7!}{3!\,2!} = 420$$

COMBINATION



REPETITION IS ALLOWED

Formula:

$$C(n+r-1,\, r) = C(n+r-1,\, n-1) = \frac{(n+r-1)!}{r!\ (n-1)!}$$

Example:

There are five flavors of ice cream: banana, chocolate, lemon, strawberry and vanilla. You can have three scoops. How many variations will there be?

Solution:

You can have 3 scoops and repetition is allowed, so it may be CCC, CCB, CCL ...etc.

By using formula;

$$C(5+3-1,\ 5-1) = \frac{(5+3-1)!}{3!\,(5-1)!}$$

$$= \frac{7!}{3!4!}$$

$$= 35 \text{ variations}$$

REPETITION IS NOT ALLOWED

Formula:

$$^{n}C_{r} = C(n,r) = \frac{n!}{r!(n-r)!}$$

Example: There are 16 balls tagged with number 1 till 16. How many ways can we pick 3 combination balls without repeating the same balls?

Solution:

$$^{n}C_{r} = C(n,r) = \frac{n!}{r!(n-r)!}$$

$$^{16}C_{3} = \frac{16!}{3!(16-3)!} = \frac{16!}{3!(13)!} = 560 \text{ combinations}$$

e.g: 123 (a combination), 345, 678, 124, 125, 126, ...,

EXERCISE 5A

1. How many ways of a student can choose a calculus professor if there are 8 male professors and 5 female professors who teach calculus class?

2. How many outfits can be made from 4 pairs of pants, 3 shirts, and 2 pairs of shoes?

3. An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

4. Make a ***tree diagram*** to answer this one. How many ways can you arrange a fun evening out if you have 3 choices for restaurants, 3 choices for movies, and 2 choices for a friend to take along? You choose the names of the movies, restaurants, and friends.

5. How many ways can you arrange 4 books on the same shelf? You can use a single letter to represent a book title (such as A, B, C, and D). (Hint: there are 4 to choose from for the first position, leaving 3 to choose from for the second position, etc.)

6. A restaurant offers a meal set where there are 5 choices of appetizer, 10 choices of main meal and 4 choices of dessert. How many different possible meals does the restaurant offer?

7. Sarah goes to her local pizza parlor and orders a pizza. She can choose either a large or a medium pizza, can choose one of seven different toppings, and can have three different choices of crust. How many different pizzas could Sarah order?

8. Derek must choose a four-digit PIN number. Each digit can be chosen from 0 to 9. How many different possible PIN numbers can Derek choose?

9. For her literature course, Rachel has to choose one novel to study from a list of four, one poem from a list of six and one short story from a list of five. How many different choices does Rachel have?

10. Jenny has nine different skirts, seven different tops, ten different pairs of shoes, two different necklaces and five different bracelets. In how many ways can Jenny dress up?

EXERCISE 5B

1. A boy lives at X and wants to go to School at Z. From his home X he has to first reach Y and then Y to Z. He may go X to Y by either 3 bus routes or 2 train routes. From there, he can either choose 4 bus routes or 5 train routes to reach Z. How many ways are there to go from X to Z?

2. A restaurant offers 5 choices of appetizer, 10 choices of the main course and 4 choices of dessert. A customer can choose to eat just one course, or two different courses, or all three courses. Assuming that all food choices are available, how many different possible meals does the restaurant offer?

   (NOTE: When you eat a course, you only pick one of the choices).

3. Suppose statement labels in a programming language can be either a single letter or a letter followed by a digit. Find the number of possible labels.

EXERCISE 5C

1. How many permutations of 3 **different** digits are there, chosen from the ten digits 0 to 9 inclusive?

2. A password consists of four **different** letters of the 26 alphabet. How many different possible passwords are there?

3. A password consists of two letters of the alphabet followed by three digits chosen from 0 to 9. Repeats are allowed. How many different possible passwords are there?

4. Assuming that any arrangement of six letters forms a 'word', how many 'words' of any length can be formed from the letters of the word SQUARE? No repeating of letters.

5. Find the number of ways that a party of seven persons can arrange themselves in a row of seven chairs.

6. Find the number of permutations that can be formed from all the letters of each word.
    a) RADAR
    b) UNUSUAL

7. In how many ways can four mathematics books, three history books, three chemistry books, and two sociology books be arranged on a shelf so that all books of the same subject are together?

8. How many vehicle plate numbers can be made if each plate contains two different letters followed by three different digits?

9. Find the number of permutations that can be formed from the letters of the word ELEVEN

10. How many of the word ELEVEN begin and end with letter E

EXERCISE 5C

1. A bag contains six white marbles and five red marbles. Find the number of ways four marbles can be drawn from the bag if
   a) They can be any color
   b) Two must be white and two red
   c) They must all be in the same color
2. John has 8 friends. In how many ways can he invite one or more of them to dinner?
3. In how many ways can a cricket-eleven be chosen out of 15 players? If
   a) A particular player is always chosen
   b) A particular player is never chosen.
4. How many committees of five with a given chairperson can be selected from 12 persons?

EXERCISE 5C

1. In a group of 11 people, 4 people are to be chosen to involve in community service. In how many ways this can be done?

2. 10 boys are running in a race. In how many different ways can the first 3 places be filled if there are no dead heats?

3. 14 students have been nominated to fill the posts of chairperson, secretary and treasurer of the Interact Club. In how many ways can the posts be filled?

4. How many different six-digits numerical can be written using all of the following six digits: 5, 5, 8, 8, 8, 7?

5. Give a possible combination for the given situations.

   a) Ayuni has 2 types of skirt, 2 blouses and 3 pairs of shoes. How many different outfit combinations of one skirt, one blouse and one pair of shoes Ayuni have?

   b) Ray restaurant has 3 types of fishes, 2 types of side dishes and 3 types of drinks. How many different meal combinations of 1 fish, 1 side dish and 1 drink that the restaurant offer?

   c) Malaysia steak house offers a 3-course meal that consists of an appetizer (onion rings, fries and popcorn), a main dish (steak, chicken, pasta) and a dessert (sundae, pie, cheese cake, chocolate cake). How many different combinations of one appertizer, one main course and one dessert the restaurant offers?

6. Find the total number of outcomes for the situation below:

   a) If a three-digit number is formed from the digits 1,2,3,4,5,6 and 7 without repetitions, calculate how many of these three-digit numbers will have a number value between 100 and 500.

   b) How many ways can 4 girls and 5 boys be arranged in a row so that all the girls are together?

   c) A door can be opened only with a security code that consists of five buttons: 1,2,3,4,5. A code consists of pressing any button, or any two, or any three, or any four, or all five. How many possible codes are there?

7. A movie theatre sells 3 sizes of popcorn (small, medium, and large) with three choices of flavours (strawberry, butter, chocolate). How many possible ways a bag of popcorn can be purchased?

8. The ice cream shop offers 31 flavours. You order a double-scoop cone. In how many different ways can the seller put the ice cream on the cone if you want two different flavours?

9. Burger Ramli offers 4 types of burgers, 5 types of beverages, and 3 types of desserts. If a meal consists of 1 burger, 1 beverage and 1 dessert, how many possible meals can be chosen?

10. You are considering 10 different colleges. Before you decide to opply to the colleges, you want to visit some or all of them. How many orders can you visit:

    i) 6 of the colleges?

    ii) all 10 colleges?

11. A group of 4 students is to be selected from a group of 10 students to take part in a class biology.

    i. In how many ways can this be done?

    ii. In how many ways can the group that will not take part be chosen?

12. How many ways of arranging 4-digit numbers from the digits 5, 6, 7 and 8 if repetition is allowed.

13. How many 3 letter words can be formed using the letters c,a,t allowing for repetition of the letters?

14. In how many ways can the letters of the word 'SIMILARLY' be arranged in a line?

15. In how many ways can the letters of the word 'DIFFERENCE' be arranged in a line?

16. In how many different ways can 4 different books be arranged on a shelf?

17. In how many different ways can 6 students be seated in a row of 6 chairs?

18. 4 chairs are arranged in arrow. In how many different ways can 6 students be seated.

19. There are a 1-sen coin, a 5-sen coin, a 10-sen coin, a 20-sen coin and 50-sen coinin a box. In how many different ways can 3 of the coins in the box be stacked?

20. In how many ways can 2 girls and 3 boys be arranged in a line if the two girls must be at the front and the back of the line respectively?

21. 9 cards are to be arranged in a row. In how many ways can all the cards be arranged if 2 particular cards must be next to each other?

22. Identify the number of arrangements for the word OPINION.

23. A committee of **6 members** is selected from a group of 4 engineers and 6 technicians. Identify the possible number of selections if the committee consists of
    a) Only 3 engineers

    b) More than 3 technicians

24. How many different arrangements can be made with all letters from the word:
    i. ARRANGEMENT

    ii. BROADBAND

25. A committee of 5 members is selected from 6 lecturers and 4 students. Find the number of ways if the committee is made up of:
    i. Three lecturer and two students.

    ii. Not more than two lecturers.

REFERENCES

1. Kevin, F. (2017). *Discrete Mathematics and Applications*(2^{nd}), Chapman and Hall (GB)
2. Rosen, K. H. (2018). *Discrete Mathematics and Its Applications* ,8th Edition. USA: McGraw-Hill Education.
3. Richard, J (2017). *Discrete Mathematics. (8^{th} Edition).* USA: Pearson
4. https://www.tutorialspoint.com/computer_logical_organization/logic_gates.htm

ISBN 978-967-0055-33-6