Meaning of proposition

A **proposition** (or **statement**) is a sentence that is either **True** or **False**.

Example of proposition

- $10 \div 2 = 4$
- 5 is an even number.
- Today is Wednesday

Example of non-proposition

- Where do you live?
- Please answer the
- question correctly.
- $x < 10$

TWO PROPOSITIONS

p	***q***
T	T
T	F
F	T
F	F

THREE PROPOSITIONS

p	***q***	***r***
T	**T**	**T**
T	**T**	**F**
T	**F**	**T**
T	**F**	**F**
F	**T**	**T**
F	**T**	**F**
F	**F**	**T**
F	**F**	**F**

CONNECTIVES IN PROPOSITION LOGIC

- CONJUNCTION $P \wedge Q$
- DISJUNCTION $P \vee Q$
- NEGATION $\sim P / \neg P / \bar{P}$
- CONDITIONAL $P \longrightarrow Q$
- BICONDITIONAL $P \longleftrightarrow$

Logical equivalence	• Two statement forms are called logically equivalent (≡) if and only if they have same truth value in every possible situation.
Tautology	• A proposition $P(p, q, r, \ldots \ldots)$ is a ***tautology*** if it contains **only T in the last column** of its truth table.
Contigency	• A proposition $P(p, q, r, \ldots \ldots)$ is a ***contingency*** if it contains both **T** and **F** in the last column of its truth table.
Contradiction	• A proposition $P(p, q, r, \ldots \ldots)$ is a ***contradiction*** if it contains **only F in the last column** of its truth table.

Type of Quantifiers

- Universal quantifier ($\forall \boldsymbol{x}$)
  $\forall$x P(x) = for every x, P(x).
- Existential quantifier ($\exists \boldsymbol{x}$)
  $\exists$x P(x) = There exist an x in P(x).

SOME EXAMPLE OF QUANTIFIERS

Let the universe be the set of airplanes and let $F(x, y)$ denote "**x** flies faster than **y**". Write each proposition in words.

a) $\forall x \forall y F(x, y)$ ➡ "Every airplane is faster than every airplane"

b) $\forall x \exists y F(x, y)$ ➡ "Every airplane is faster than some airplane"

c) $\exists x \forall y F(x, y)$ ➡ "Some airplane is faster than every airplane"

d) $\exists x \exists y F(x, y)$ ➡ "Some airplane is faster than some airplane"

VALIDITY OF ARGUMENT

An argument is said to be ***valid*** if $\boldsymbol{Q}$ is true whenever all the premises $\boldsymbol{P_1}$, $\boldsymbol{P_2}$, …, $\boldsymbol{P_n}$ are true.

TEST THE VALIDITY USING TRUTH TABLE.

Example:

Show that the following argument is valid or fallacy.

a)
$$\begin{array}{l} p \to q \\ p \\ \hline \therefore\ q \end{array}$$

Solution:

p	q	$\boldsymbol{p \to q}$	p	q
T	T	T	T	**T**
T	F	F – ignore!	T	–
F	T	T	F – ignore!	–
F	F	T	F – ignore!	–

VALID

RULE OF INFERENCE	TAUTOLOGY	NAME
$\boldsymbol{p \to q}$ $\boldsymbol{p}$ $\boldsymbol{\therefore q}$	$[\boldsymbol{p} \wedge (\boldsymbol{p} \to \boldsymbol{q})] \to \boldsymbol{q}$	**Modus ponens**
$\boldsymbol{p \to q}$ $\boldsymbol{\neg q}$ $\boldsymbol{\therefore \neg p}$	$[\neg \boldsymbol{q} \wedge (\boldsymbol{p} \to \boldsymbol{q})] \to \neg \boldsymbol{p}$	**Modus tollens**
$\boldsymbol{p \to q}$ $\boldsymbol{q \to r}$ $\boldsymbol{\therefore p \to r}$	$[(\boldsymbol{p} \to \boldsymbol{q}) \wedge (\boldsymbol{q} \to \boldsymbol{r})] \to (\boldsymbol{p} \to \boldsymbol{r})$	**Hypothetical syllogism**
$\boldsymbol{p \vee q}$ $\boldsymbol{\neg p}$ $\boldsymbol{\therefore q}$	$[(\boldsymbol{p} \vee \boldsymbol{q}) \wedge \neg \boldsymbol{p}] \to \boldsymbol{q}$	**Disjunctive syllogism**
$\boldsymbol{p}$ $\boldsymbol{\therefore p \vee q}$	$\boldsymbol{p} \to (\boldsymbol{p} \vee \boldsymbol{q})$	**Addition**

TEST THE VALIDITY USING TABLE RULES OF INFERENCE.

Alice is mathematics major. Therefore, Alice is either mathematics major or a computer science major.

Solution:

Identify the premise:

p: Alice is mathematics major.

q: Alice is a computer science major

Check the given statement:

p (Alice is mathematics major)

$\boldsymbol{\therefore p \vee q}$

(Therefore, Alice is either mathematics major or a computer science major)

Refer to the rules of inference:

Addition