

Meaning of proposition

A **proposition** (or **statement**) is a sentence that is either **True** or **False**.

Example of proposition

- $10 \div 2 = 4$
- 5 is an even number.
- Today is Wednesday

Example of non-proposition

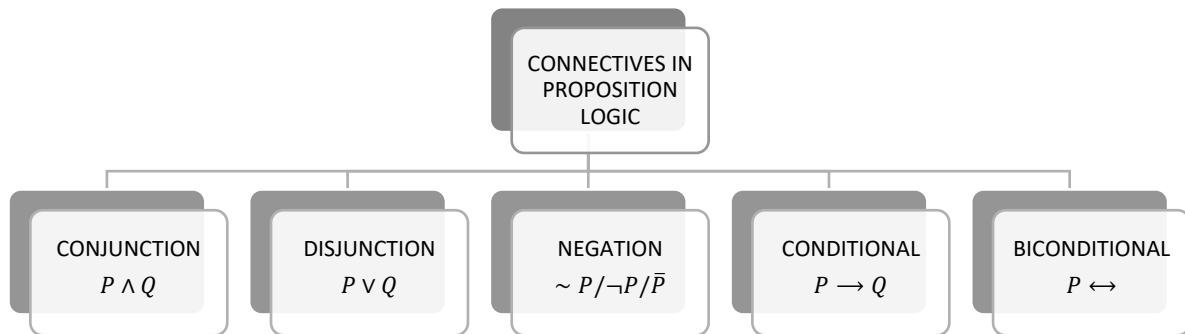
- Where do you live?
- Please answer the question correctly.
- $x < 10$

TWO PROPOSITIONS

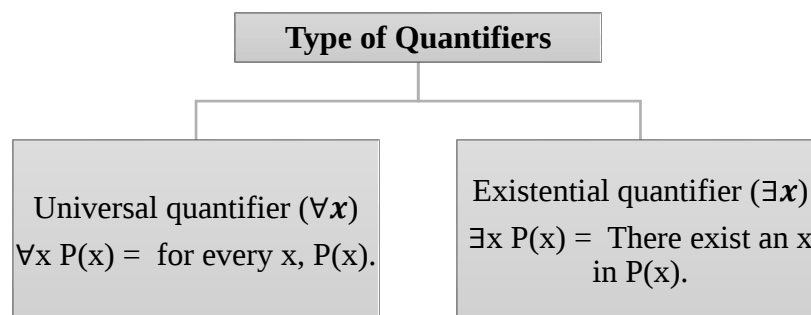
<i>p</i>	<i>q</i>
T	T
T	F
F	T
F	F

THREE PROPOSITIONS

<i>p</i>	<i>q</i>	<i>r</i>
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F



Logical equivalence	Two statement forms are called logically equivalent ($\equiv$) if and only if they have same truth value in every possible situation.
Tautology	A proposition $P(p, q, r, \dots)$ is a tautology if it contains only T in the last column of its truth table.
Contingency	A proposition $P(p, q, r, \dots)$ is a contingency if it contains both T and F in the last column of its truth table.
Contradiction	A proposition $P(p, q, r, \dots)$ is a contradiction if it contains only F in the last column of its truth table.



SOME EXAMPLE OF QUANTIFIERS

Let the universe be the set of airplanes and let $F(x, y)$ denote “ x flies faster than y ”. Write each proposition in words.

- a) $\forall x \forall y F(x, y)$ $\longrightarrow$ “Every airplane is faster than every airplane”
- b) $\forall x \exists y F(x, y)$ $\longrightarrow$ “Every airplane is faster than some airplane”
- c) $\exists x \forall y F(x, y)$ $\longrightarrow$ “Some airplane is faster than every airplane”
- d) $\exists x \exists y F(x, y)$ $\longrightarrow$ “Some airplane is faster than some airplane”

VALIDITY OF ARGUMENT

An argument is said to be **valid** if Q is true whenever all the premises $P_1, P_2, \dots, P_n$ are true.

TEST THE VALIDITY USING TRUTH TABLE.

Example:

Show that the following argument is valid or fallacy.

$$\begin{array}{l} \text{a)} \quad p \rightarrow q \\ \quad \quad p \\ \hline \quad \quad \therefore q \end{array}$$

Solution:

p	q	$p \rightarrow q$	p	q
T	T	T	T	T
T	F	F – ignore!	T	–
F	T	T	F – ignore!	–
F	F	T	F – ignore!	–

VALID

RULE OF INFERENCE	TAUTOLOGY	NAME
$\frac{p \rightarrow q \quad p}{\therefore q}$	$[p \wedge (p \rightarrow q)] \rightarrow q$	Modus ponens
$\frac{p \rightarrow q \quad \neg q}{\therefore \neg p}$	$[\neg q \wedge (p \rightarrow q)] \rightarrow \neg p$	Modus tollens
$\frac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$\frac{p \vee q \quad \neg p}{\therefore q}$	$[(p \vee q) \wedge \neg p] \rightarrow q$	Disjunctive syllogism
$\frac{p}{\therefore p \vee q}$	$p \rightarrow (p \vee q)$	Addition

TEST THE VALIDITY USING TABLE RULES OF INFERENCE.

Alice is mathematics major. Therefore, Alice is either mathematics major or a computer science major.

Solution:

Identify the premise:

p : Alice is mathematics major.

q : Alice is a computer science major

Check the given statement:

p (Alice is mathematics major)

$\therefore p \vee q$

(Therefore, Alice is either mathematics major or a computer science major)

Refer to the rules of inference:

Addition